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DEFINITION OF TIME COMPONENT PETRI NET FOR DIFFERENT WAYS OF IT CONSTRUCTION

In article features of creation of a component Petri net with time characteristics are considered. Definitions for each possible way of construction of the temporary component model are formulated.

Key words: Component Petri net with time, component modeling, structural delay, behavioral delay

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Introduction

Creation of difficult industrial systems required not only for careful designing of working blocks and their communications, but also for rational management of necessary resources. Time is one of such resources and it's optimum use can lead to the essential reduction in price of manufacture. The theory of Petri nets [1], [2] allows to create and verify models with the true parallelism, what is extremely important for the modern computing systems. The extension of the existing Petri nets theory by adding of temporal property, allows to reflect more in detail dynamics of functioning of the researched task. But thus, because of explosive character in growth of the size of a state graph, formal verification becomes complicated considerably.

In search of a solution of *state explosion problem* at verification of Petri nets, the theory of component modeling and analysis of Petri nets has been created. Which basic property to "show"difficult, by the functional point of view, and the same type parts of an initial Petri net, facilitating and accelerating of verification [3], [4]. This is the result of the primary goal of component modeling — selection of compound component, what allows to receive the reduced model (concerning initial detailed), being adequate to initial investigated system.

Addition of the time characteristic to component modeling, will allow to receive more capacious representation of models of real time systems. After which it is possible to order in time the events of modeled structure (i.e. drawing up of schedules) already at an analysis stage of the component nets. Introduction of temporary restrictions probably at different stages of creation of the time component model. Namely: allocation a component in time model or introduction of the concrete time construction in existing component model.

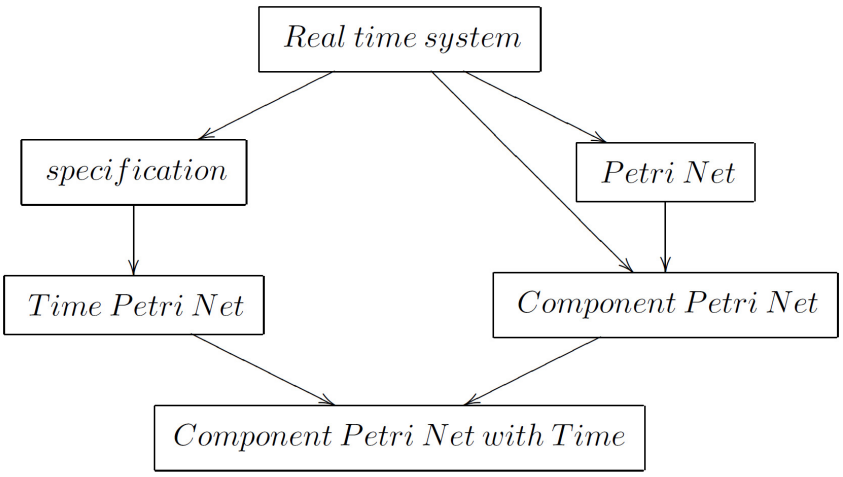
Specificity and properties of temporal extensions of Petri nets is formed by singularities of the specific researched systems, purpose setting at modeling and also existing methods of verification concrete and abstract models. There are [5] some basic approaches to introduction of the temporary characteristics in the device of modeling by Petri nets. The first of them consists in assigned clocks to the net's various elements(to the places [6], to the transitions [7], or to the processes [5]), whereas the second one uses so-called firing intervals [8], time delays of the transitions [12] or availability of labels [9]. Each of these approaches has many nuances.

The aim of the article is reviewing of singularities of introduction of the temporal characteristic in the component Petri net theory. And also the description of possibilities of the component analysis at various stages of creation of model of initial system with time.

Problem statement

Component Petri net (CN-net), as the modeling structure, has the important singularities. Places and transitions in a CN-net can be various types: finite sets of places and transitions include the subsets of composite components(components-places C_p and components-transitions C_t). Functioning of composite components in a CN-net is understood as the instant performance that allows to ignore internal operation of a composite component at this level of the model. The instant operation of the compound components for CN-net, for components causes it's being in an active condition some time [10], in what finds the display of two-aspect approach to functioning of compound components. This approach is used at realisation of the component analysis of a detailed Petri net and, as consequence, only one of representatives of the same components is checked [3].

Introduction of the explicit time characteristic in the theory of component modeling means deviation from, traditional for the general theory of Petri nets, understanding of instant operation of transitions. Thus, to expand component model with time designs probably in the various ways: an association clocks to places , an association clocks to transitions, by use of firing intervals of transitions, by introduction time delays for transitions of a net. In given article used the approach, which will associate time delays to transitions of the Petri net. The formal description of the given approach is resulted



Pic. 1. Different ways of Time component Petri net construction

in work [12], we will use it for a case of Petri nets with single-channel transitions. The explicit advantage here consists in independent verification of structural and temporal properties already on the model constructed in terms of time Petri nets [13].

Addition of a time condition will be carried out in two ways:

(1) introduction of the time restrictions in the component Petri net, constructed on a detailed Petri net of investigated system or at once for investigated system;
(2) allocation composite components in a Petri net, described in terms of time Petri net.

The scheme presented in picture1, reflects these various ways of construction of CN-model with time characteristics.

Following logic of this scheme, we will result formal definition of a component net in due course, which will be different for various ways of construction of model of real time system. The expediency of uniform definition will be considered.

Time inclusion in a CN-net

The component Petri net is a tuple (P, T, F, W, D, M_0), where $P = P_1^* \bigcup P_2$ — finite set of places, $T = T_1^* \bigcup T_2$ — finite set of the transitions, understood in terms of component Petri net (P_1^* and T_1^* accordingly finite sets of composite-places and composite-transitions, P_2 и T_2 — accordingly finite sets of places and transitions, understood in normal sense of places and transitions of Petri net, remained after selection of composite-places and composite-transitions), $F \subseteq P \times T \bigcup T \times P$ — the flow relation between places and transitions, $W : F \to N \setminus \{0\}$ — frequency rates of arcs function, M_0 — an initial marking of a net [11].

The flow relation F and function of frequency rate of arches W define flow function$I$, setting a rule $I : (P \times T) \bigcup (T \times P) \to N$ and defining that elements of one set cannot be connected by arches, and also describing sets of entrance and target elements.

Let's consider CN-net functioning in the discrete time, consisting of the equal intervals (steps) numbered by integers. And if the condition of a component net is described unequivocal by marks of places, division in time of starts and ending operations of transitions in time model, brings additional variability (as consequence, growth of space of states of a system occurs like explosion). Hence, under a state of the time component net we will understand the concrete moment of functioning of the given net which will be described state of places (marking) and a state of transitions (history of starts of transition, during time, not exceeding it's maximum duration of operation [13]).

In a CN-net there can be either composite-transitions (C_t), or composite-places (C_p), or both simultaneously. For the component net in due course we will enter designation CN_t, without dependence from the type of allocated component.

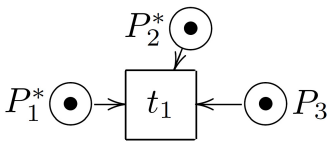
Introduction of the time characteristic in a CN-net only with composite-transitions. Addition of time delays to transitions of a CN-net, demands consideration the understanding of time of operation of transitions, internal for component C_t. Following variants are possible:

(1) all internal transitions of component will necessarily finish the work in given to C_t time ("the schedule"approach). Analysis of properties of composite-transition will separate time by proportional parts, which appointed between all transitions forming it's structure;

(2) time associated to a composite-transition, will be considered as the assumption that it's internal transitions will have time to fulfil in due time ("the assumption"approach). Given to C_t component time can be too much or not suffice for work of all it's internal transitions — that is subject to check at verification properties of the components.

Characteristic for the first variant of understanding of time operation of composite components is orientation on finding-out of structural properties of a net, for the second — finding-out of time properties. In the consecutive application of these two considerations at first to a CN_t-net, and then to separated compound components, finds the representation of the two-aspect approach to understanding of functioning of a CN_t-net.

Definition 1. Component Petri net with time characteristics (CN_t-net), containing only composite-transitions, is a tuple (P, T, F, W, D, M_0), where P — finite set of places;$T = T_1^* \bigcup T_2$ — finite set of transitions (T_1^* — finite set of composite-transitions); $F \subseteq (P \times T) \cup (T \times P)$ — the flow relation. Display $W : F \to \{1, 2, \ldots\}$ defines number of arches connecting places and transitions, temporary map $D : T \to \{1, 2, \ldots\}$ sets times of



Pic. 2. Transition connected by incidence arches with several components-places

operations of transitions of a net, out of dependences on their type. M_0 — an initial marking of a net.

Introduction of the time characteristic in a CN-net only with composite-places. Associating of temporary restrictions to transitions of the CN-net containing only components-places, attracts some feature: timing delays also will be appropriate to internal transitions of components-places, but how and on what stage will be presented time delay for the graph of a component time Petri net, if time properties associates only to transitions? After all, unlike a non-temporary net, in which operation time of C_p vaguely, time spent in a composite-place of a CN_t-net , will have the concrete component. Means, two alternative approaches are possible:

(1) Concrete operation time of C_p is described obviously on the graph, thus the logic of associating time restrictions to transitions remains.
(2) Concrete operation time of internal transitions is not displayed obviously for composite components on the graph, for the purpose of preservation of an initial formalism of uncertainty of stay a tokens in places of a Petri net.

    And, implicit representation of a time delay of composite components on the graph of a net, can be expressed by increase in a time delay of the output transition (transitions) for components C_p:
    (a) by known time operation of composite-places,
    (b) by some preliminary (parametrized) value of time of actuating component of this kind,
    (c) it is kept invariable (a convenient variant at creation of model of time system, when time frameworks of separate working blocks are not yet known).

It is required to specify value which will be increase the concrete time delay of the transition connected by incidence arches with several components-places (picture 2) for variants $2a$, $2b$.

Activating of the transition will happens only when in all entrance places will be tokens, and their number will be equal to value of frequency rates of arcs function W. After moving of tokens in entrance places of composite components, internal for components-places C_p transitions can work:

- In parallel — simultaneous operation of the maximum number of internal transitions various composite-places.
- Consecutive operation of internal transitions of various composite-places probably, at existence of any long delays of tokens in places C_p.
- Independent operation of internal transitions of various composite-places means them and parallel, and consecutive work.

So, if the way of functioning parallel composite components is not known, their common time delay will be represented by a time interval which bottom border will be correspond to parallel operation of internal transitions of composite-places, and top border — to consecutive operation of internal transitions of composite component C_p. Means, time delay of a transition, which is output for several components C_p, will be increase on their common time delay or on average value from an interval of common operation. The bottom border of an added time interval can be calculated as the longest operations time of one of the parallel composite component, and top border — the sum of times of all the functioning transitions, which are internal for parallel compound component.

The variant 2c obviously is easier previous and represents advantage of the two-aspect approach of Component Modeling. Consideration of time properties of C_p-components will be carried out at a stage of finding-out it's properties, i.e. for a CN^t-net time of composite-places will be neither displayed (obvious 1 or indirect 2a, 2b way), nor investigated.

Hence, a CN^t-net exclusive with composite-places can be defined as follows:

Definition 2. Component Petri net with time characteristics , containing only composite-places, is a tuple (P, T, F, W, D, M_0), where $P = P_1^* \bigcup P_2$ — finite set of places (P_1^* — finite set of composite-places); T — finite set of transitions; $F \subseteq (P \times T) \cup (T \times P)$ — the flow relation. M_0 — an initial marking of a net. Display $W : F \to \{1, 2, \ldots\}$ defines frequency rate of arches connecting places and transitions. Temporary map D will look like:

- For the approach 1 $D : T, P_1^* \to \{1, 2, \ldots\}$.
- For the approaches 2a и 2b $D : T \to \{t_k + [t_l, \sum_{t_l \in P_m^* F t_k} t_l]\}$, where t_k — concrete temporal delay of the transition, $k = \overline{1, n}$, where n — number of transitions of CN-net, $m = \overline{1, s}$, where s — number of composite-places of CN-net
  - for 2a t_l — known concrete time delay of the composite-place.
  - for 2b t_l — prospective concrete time delay of the composite-place.
- For the approach 2c $D : T \to \{1, 2, \ldots\}$.

Introduction of time characteristic in a CN-net with components of both types. Addition of time properties in a CN-net possessing composite-places and composite-transitions, will carried out by following stages:

(1) Time delays associate to CN-net transitions, without dependence from their type,
(2) The concrete time delay peculiar for composite-place C_p increases operation time for it's output transition (transitions) and it is not described on the graph (variants 2a, 2b or 2c). It can be a composite-transition for output of composite-places, the rule doesn't change.

Thus, using approaches of "schedule"and "assumption"in understanding of operation time of transitions of all types, probably to design models of systems with set time of functioning working blocks or to investigate time properties of systems with certain structure.

Definition 3. Component Petri net with time characteristics is a tuple (P, T, F, W, D, M_0), where $P = P_1^* \bigcup P_2$ — finite set of places; $T = T_1^* \bigcup T_2$ — finite set of transitions (P_1^* и T_1^* accordingly finite sets of composite-places and composite-transitions); $F \subseteq (P \times T) \cup (T \times P)$ — flow relation. M_0 — an initial marking of a net. Map $W : F \to \{1, 2, \ldots\}$ defines a number of the arcs connecting places and transitions. Temporary map D will look like:

- For the approaches 2a и 2b $D : T \to \{t_k + [t_l, \sum_{t_l \in P_m^* F t_k} t_l]\}$, where t_k — concrete temporal delay of the transition, $k = \overline{1, n}$, where n— number of transitions of CN-net, $m = \overline{1, s}$, where s— number of composite-places of CN-net
  - for 2a t_l — known concrete time delay of the composite-place.
  - for 2b t_l — prospective concrete time delay of the composite-place.
- For the approach 2c $D : T \to \{1, 2, \ldots\}$.

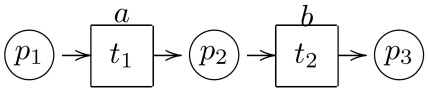
Selection composite components in a time Petri net

Time Petri Net is a tuple (P, T, F, W, D, M_0), where $P = \{p\}$ — finite set of places, $T = \{t\}$ — finite set of transitions; $F \subseteq (P \times T) \cup (T \times P)$ — flow relation. Map $W : F \to \{1, 2, \ldots\}$ defines a number of the arcs connecting places and transitions, $D : T \to \{1, 2, \ldots\}$ sets operations times of transitions of a net, M_0 — initial marking of a net.

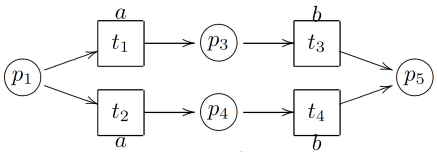
Multiplicity of the arcs $(p, t) \in F$, $(t, p) \in F$ and operations times of transitions $t \in T$ will be Accordingly designated $w_{p,t}$, $w_{t,p}$, d_t.

Let's select composite components so that the size of a net reduce as much as possible. In spite of the fact that verification of composite-transitions is easier than verification of composite-places, selection of the last is in certain cases more advantageous from the point of view of verification simplification for all CN-model. Therefore we will select composite components in a time Petri net, without rejecting possibility of selection composite component of other kind.

We will select section of the time Petri Net as a composite component $C_p(C_t)$, if it's starting and finishing with place or places (one or several transitions), not having the



Pic. 3. Elementary composite component of Petri net



Pic. 4. Composite component of Petri net with parallel transitions

flow arches connecting another parts of the net and internal places or transitions of this section. If there are section with just the same structure in a detailed Petri net, they will be selected as compound composite-places (composite-transitions) with the same name $P_i(T_i)$. If the structure of a part of a net differs from structure of already selected composite-places (composite-transitions) by number of parallel processes, we select the same type composite-place (the same type composite-transition) and sign it accordingly $P_i^*(T_i^*)$. The parts of a net selected as the same type composite-places (the same type composite-transitions) presented on pictures 3 and 4. As a result of replacement of parts of a detailed time Petri net with places P_i, P_i^* and transitions T_i, T_i^* we will have a component time Petri net.

Let's consider now a question of the description of the time delays peculiar to composite components selected in time Petri net.

Calculation of the time delays peculiar to composite components of the CN_t-net. Time delay peculiar to selected composite component of any type, will depend from:

- the structure create, formed by
  - amount of places and transitions in composite component,
  - presence of parallel transitions in composite component,
  - duration of the time delays of the initial for composite component transitions.
- the behavioural create, formed by
  - amount of tokens, going through the composite component;
  - size of the indefinite delay in places of the net.

To calculate a concrete time delay for composite component of any type probably only with known both structural, and behavioural the forming.

Example. There is elementary component-place presented at picture 3. It's structural time delay, obviously, will be received by addition of time delays of all internal transitions (t_1 and t_2 — $a + b$). The general time delay depending both from behavioural, and from the structural forming, will be calculated:

- at simultaneous arrival of several tokens in a component-place:

  - if the time delay of transitions $a \leq b$, then the general time delay of a component will be equal $a + b \cdot k$, where k — number of the tokens sequentially transiting a composite component.
  - if the time delay of transitions $a > b$, then the general time delay of a component will be equal $a \cdot k + b$, where k — number of the tokens sequentially transiting a composite component.
- at consecutive arrival of several tokens in a component-place with an interval δ
  - if an inflow interval between tokens $b \leq \delta \leq a$, a tokens will go one for another without interval since transitions one-channel. Hence the general time delay for a component will be same as well, as at simultaneous arrival of tokens in the component (look point above).
  - if an inflow interval between tokens $a \leq \delta \leq b$, then the general time delay will be $a \cdot k + b \cdot (k-1) + \delta$, where k — number of the tokens transiting a composite component.
  - if an inflow interval between tokens $\delta + a \leq b$, then the general time delay will be $a + b \cdot k + \delta$, where k — number of the tokens transiting a composite component.
  - at strongly removed from each other in time inflow of several tokens in a component-place — i.e. $\delta \geq (a + b)$ $(a + b) \cdot k$,where k — number of the tokens transiting a composite component.

For the composite component presented in picture 4, the structural time delay will be $a + b$. (But already at other duration of transition, for example the time delay of t_3 will be c instead of b, structural time delay will be indefinite and differ from the passed way — $a + b$ or $a + c$.) And the general time delay, at the most simple case of simultaneous inflow of tokens into the composite component and their uninterrupted movement will be:

- at a ratio of time delays of transitions $a < b$, then the general time delay will be $a \cdot k + b$, $k = 1, 2, \ldots$ for amount of tokens from 1 till 2 for $k = 1$, for $k = 2$ amount of tokens is 3 or 4 and e.t.c.
- at a ratio of time delays of transitions $a \geq b$, then the general time delay will be $b \cdot k + a$, $k = 1, 2, \ldots$ for amount of tokens from 1 till 2 for $k = 1$, for $k = 2$ amount of tokens is 3 or 4 and e.t.c.

Consideration of a case of arrival of tokens with some interval δ, will demand comparison of sizes of time delays of transitions with duration of δ and various combinations of their common duration for several tokens. What will be enough labour-intensive process even for the elementary on structure composite-place.

Hence, we will calculate a time delay of composite components after finding of behaviour properties of CN-net, when the maximum quantity of passing through tokens becomes known.

And as the theory of component modeling allows to verify properties only one of representatives of same type components, and we can know properties of the others from theorems [3]. Time properties of composite components also would be convenient for calculating on the basis of one formula, which would be peculiar to composite components of certain structure.

The two-aspect approach apply to the time functioning of composite components of CN_t-model will result in not representing of time delay of composite-places C_p and/or composite-transitions C_t. Consideration of time properties will be carried out first for the composite components, and after for the initial, not composite model.

Definition 4. Component Petri net with time characteristics (CN_t-net), containing only composite-transitions, is a tuple (P, T, F, W, D, M_0), where $P = P_1^* \bigcup P_2$ — finite set of places; $T = T_1^* \bigcup T_2$ — finite set of transitions (P_1^* and T_1^* — finite sets of composite-places and composite-transitions); $F \subseteq (P \times T) \cup (T \times P)$ — the flow relation. Display $W : F \to \{1, 2, \ldots\}$ defines number of arches connecting places and transitions, temporary map $D : T_2 \to \{1, 2, \ldots\}$ sets times of operations of non composites transitions of a net. M_0 — an initial marking of a net.

Conclusions

Mathematical model of time Petri net containing subsets of places and the transitions of a special kind named composite components formulated in paper . This allows to use advantages of component Petri net theory to effective research of models of systems with time in due course. Singularities of work of the component model arising from adding of temporary feature are considered. In case of creation of component model on the basis of system with time, constructed the examples illustrating necessity of two-aspect approach of component modeling for a solution of "state explosion"in case of temporary Petri nets.

Following stage of work in given direction seen in researching of temporary properties of composite components of certain structure. The ways of finding of temporary properties of component models also represent certain practical interest.

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Определение временной компонентной сети Петри для различных путей ее построения *В статье рассматриваются особенности создания компонентной сети Петри с временными характеристиками. Формулируются определения для каждого возможного пути построения временной компонентной модели.*

Ключевые слова: Компонентная сеть Петри со временем, компонентное моделирование, структурная задержка, поведенческая задержка

Означення компонентної сеті Петрі із часом для різноманітних шляхів конструювання *У роботі розглянуті властивості конструювання компонентої сеті Петрі із часовою характеристикою. Сформульовані означення для кожного із шляхів конструювання компонентої моделі з часом.*

Ключові слова: Компонентна сеть Петрі із часом, компонентне моделювання , структурна затримка, затримка поведіння