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FIVE-DIMENSIONAL GEOMETRY OF SUPERMULTIVERS AS A RESULT OF THE SYNERGETIC SELF-ORGANIZATION OF STOCHASTIC FLUCTUATIONS IN THE MINKOWSKI GEOMETRY

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This work is the natural synthesis of two modern cosmological theories: five-dimensional cosmological theories that arise as the development of a unified field theory proposed T. Kaluza, and the theory of eternal inflation Guth-Linde-Vilenkin. The theory of eternal inflation does not give an answer to the question of why the "inflaton" has given, and not some other form of its potential energy. Our approach is based on the variety of geometry as a dynamic system, and the role played by the time the 5th coordinate. The dynamics of such a system is described by equation of E. Cartan, the solution of which defines the geometry in our 5-dimensional manifold. This equation is first stochastically quantized at G. Haken and then turns into a nonlinear using the ideas of Yu. Klimontovich. The resulting nonlinear generalization of the Fokker-Planck equation describes the self-organization of fluctuations in the Minkowski space. Moreover, the fluctuations at different values of the 5th coordinates can self-organize in the geometry generated by different energy-momentum tensor of "inflaton" with a different structure of the potential energy.

Keywords: inflaton, universe, fluctuations, self-organization, Hilbert-Einstein equations, energy-momentum.

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INTRODUCTION

Modern cosmology regards our universe as one of the innumerable universes which form the multiverse.

One of the most well-known mathematical models of the multiverse theory of eternal inflation is a Guth-Vilenkin-Linde. According to this theory, our multiverse occurs as a quantum fluctuation creating from "nothing" 3-dimensional pseudosphere a certain (small) radius. Originating it begins to expand under the influence of "dark energy"-inflaton-scalar field, the value of which corresponds to the maximum of its potential energy (the false vacuum state). When in different areas of the multiverse inflaton rolls down from a peak to a minimum of its potential energy (the true vacuum state), the difference in potential energy of inflaton transform into the energy of the nascent high-temperature cluster of elementary particles. Each such "big bang" leads to "island universe" – our own and other similar (A. Guth, A. Linde, A. Vilenkin) [1-3]. The disadvantage of this approach is that it is not clear why the geometry of space-time must satisfy the Einstein equation with inflaton which having this structure. Who chose this structure ?

In this paper we attempt to solve the problem of inflaton origin on the basis of the 5-dimensional cosmology. This kind of cosmology emerged as the development of T. Kalutza, which introduced a pseudo-Riemannian geometry around the five-dimensional space. Einstein proposed to move from metric geometry to the geometry of spaces of affine connection, which would include both electromagnetic and gravitational fields. A unified field theory was constructed in the article of Yu. Mendygulov and I. Selezov [4], where a 5-dimensional space was introduced electro-gravitational connectivity. However, the geometry of all these studies was cylindrical in 5th coordinate.

In the late 20th century and early 21th century there was a lot of work on the 5-dimensional cosmology, which uses no cylindrical solutions of 5-dimensional Hilbert-Einstein equations [5].

We offer a totally new approach to the 5-dimensional cosmology, based on a generalization of the Hilbert-Einstein equations that describes the self-organization of the fluctuations of the Minkowski geometry in the geometry of curved four-dimensional space-time, taking place on the 5th coordinate x^4. Arising in this way 4-geometry of the 4-dimensional sections 5-dimensional space described by the standard Hilbert-Einstein equations, energy-momentum tensor of which is the energy-momentum tensor of the scalar field (inflaton) with a definite form of potential energy. Thus, a 5-dimensional space there is a set of 4-dimensional cross-sections (space-times), each of which is a multiverse of Guth-Vilenkin-Linde, the geometry of which is the result of Darwinian selection of modes of stochastic fluctuations of 4-dimensional geometries in the transition from one 4 -dimensional cross-section to another.

1. BASIC EQUATIONS

The construction of the desired generalization of Einstein's equation we begin with a description of stochastic fluctuations in the geometry of Minkowski space, presenting curvature of the Riemannian connection $R_{\mu\nu}$ as a dynamic system of mechanics Cartan [6]. To do this, first we assume that the Hilbert-Einstein equations of Minkowski space

$$R_{\mu\nu} = 0, \text{ where } \mu, \nu = 0, 1, 2, 3, \tag{1}$$

is an equation that describes the steady-state solution of the equation:

$$dR_{\mu\nu} / dx^4 = -R_{\mu\nu} \quad , \mu, \nu = 0, 1, 2, 3. \tag{2}$$

In the 5-dimensional differentiable manifold with a system of local coordinates (x^{μ}, x^4), 4-dimensional sections that x^4 = const describe the space-times of the various multiverse.

In mechanics, the equations of the type (2) may take the form of Cartan equations:

$$\frac{\delta\Omega_0}{\delta d\widetilde{R}_{\mu\nu}} = 0 \ , \tag{3}$$

where Ω_0 is the symplectic metric

$$\Omega_0 = \int \sqrt{-g}dx^0 \Lambda ... \Lambda dx^3 \left\{ d\widetilde{R}_{\mu\nu} \Lambda \left(dR^{\mu\nu} + R^{\mu\nu} dx^4 \right) \right\}, \tag{4}$$

here $\mu, \nu = \overline{0,3}$, $g = \det\left\| g_{\alpha\beta} \right\|$. We see that in our approach to the Hilbert-Einstein 5-manifold plays the role of the extended phase space of equation (2).

Now, to describe the fluctuations of the Minkowski geometry, we introduce the auxiliary variables $\Theta_{\mu\nu}$ and $\Theta_{\mu\nu\alpha\beta}$ as well as random forces $dW^{\mu\nu}$ and pass from the symplectic metric (4) to the symplectic metric:

$$\Omega_1 = \int \sqrt{-g}dx^0 \Lambda ... \Lambda dx^3 \{ d\tilde{R}_{\mu\nu}(x) \Lambda \left(dR^{\mu\nu}(x) + R^{\mu\nu}(x)dx^4 - dW^{\mu\nu}(x) \right) + \left\langle dW^{\mu\nu}(x) \right\rangle \Lambda d\Theta_{\mu\nu}(x) +$$
$$+ \int \sqrt{-g}dy^0 \Lambda ... \Lambda dy^3 \left[\left(\left\langle dW^{\mu\nu}(x) dW^{\alpha\beta}(y) \right\rangle - \delta(x-y) g^{\mu\alpha}(x) g^{\nu\beta}(y) Q dx^4 \right) \Lambda d\Theta_{\mu\nu\alpha\beta}(x,y) \right] \}, \tag{5}$$

Averaging is understood in the sense $\langle ... \rangle = \int DdW(x)\rho[dW(x)]...$, here Q is the square of the amplitude of fluctuations of the random component of force $dW^{\alpha\beta}$.

This symplectic metric also describes the world of Minkowski, but with fluctuating geometry. Indeed, the equations of Cartan [6]

$$\frac{\delta\Omega_1}{\delta d\tilde{R}_{\mu\nu}(x)} = \frac{\delta\Omega_1}{\delta d\Theta_{\mu\nu}(x)} = \frac{\delta\Omega_1}{\delta d\Theta_{\mu\nu\alpha\beta}(x,y)} = 0, \tag{6}$$

give:

$$\begin{aligned} & dR^{\mu\nu}(x) + R^{\mu\nu}(x)dx^4 - dW^{\mu\nu}(x) = \left\langle dW^{\mu\nu}(x) \right\rangle = \\ & = \left\langle dW^{\mu\nu}(x) dW^{\alpha\beta}(y) \right\rangle - \delta(x-y) g^{\mu\alpha}(x) g^{\nu\beta}(y) Q dx^4 = 0. \end{aligned} \tag{7}$$

The system of equations (7) – is a system of Ito stochastic equations and describes the statistical properties of the random force (G. Haken) [7] and as time stands 5-th coordinate. Thus, we see that the mechanics of the Cartan can describe not only the conventional continual dynamic system, but also stochastic. The consequence of (7) is the equation:

$$\frac{d\left\langle R^{\mu\nu} \right\rangle}{dx^4} = -\left\langle R^{\mu\nu} \right\rangle. \tag{8}$$

It is analogous to equation (2), but describes the mean curvature at this point a 5-dimensional space. If the curvature is not dependent on the 4-dimensional cross section, the result of equation (8) is a condition

$$\left\langle R^{\mu\nu} \right\rangle = 0 \ , \tag{9}$$

which must be satisfied in this case, the Minkowski space with a fluctuating metric. Especially strongly fluctuating geometry at the Planck scale, where it is a quantum foam (Misner, Thorne, Wheeler) [8]. Ito equation (7) is equivalent to the Fokker-Planck equation

$$\frac{df\left[R^{\mu\nu}(x)\right]}{dx^4} = \int \sqrt{-g}dx^0 \Lambda ... \Lambda dx^3 \left\{ \frac{\delta}{\delta R^{\mu\nu}(x)} R^{\mu\nu}(x) + \frac{Q}{2} \frac{\delta^2}{\delta R^{\mu\nu}(x) \delta R_{\mu\nu}(x)} \right\} f\left[R^{\mu\nu}(x)\right] \tag{10}$$

For the proof we consider following H. Haken [7], the derivative of the x^4 coordinate 4-dimensional space-time sections of our 5-dimensional manifold mean any functional $\Phi[R^{\mu\nu}]$:

$$\frac{d}{dx^4}\left\langle \Phi\left[R^{\mu\nu}\right]\right\rangle = \int DR\frac{df\left[R^{\mu\nu}\right]}{dx^4}\Phi\left[R^{\mu\nu}\right] = \left\langle \frac{d\Phi\left[R^{\mu\nu}\right]}{dx^4}\right\rangle = \left\langle \frac{1}{dx^4}\left\{\int \sqrt{-g}\,dx^0\Lambda...\Lambda dx^3 dR^{\mu\nu}(x)\frac{\delta\Phi}{\delta R^{\mu\nu}(x)} - \right.\right.$$

$$\left.\left. -\frac{1}{2}\int g dx^0\Lambda...\Lambda dy^3\frac{\delta^2\Phi}{\delta R^{\mu\nu}(x)\delta R^{\alpha\beta}(y)}dR^{\mu\nu}(x)dR^{\alpha\beta}(y) + ...\right\}\right\rangle = \left\langle \frac{1}{dx^4}\left\{\int \sqrt{-g}\,dx^0\Lambda...\Lambda dx^3\left(-R^{\mu\nu}dx^4 + \right.\right.\right.$$

$$\left.\left.\left. + dW^{\mu\nu}\right)\frac{\delta\Phi}{\delta R^{\mu\nu}(x)} - \frac{1}{2}\int g dx^0\Lambda...\Lambda dy^3\left(-R^{\mu\nu}dx^4 + dW^{\mu\nu}\right)\left(-R^{\alpha\beta}dx^4 + dW^{\alpha\beta}\right)\frac{\delta^2\Phi}{\delta R^{\mu\nu}(x)\delta R^{\alpha\beta}(y)}\right\}\right\rangle =$$

$$= \int DRf\left[R^{\mu\nu}\right]\left(-\int \sqrt{-g}\,dx^0\Lambda...\Lambda dx^3 R^{\mu\nu}(x)\frac{\delta\Phi}{\delta R^{\mu\nu}(x)} - \frac{1}{2}\int g dx^0\Lambda...\Lambda dy^3\left\langle\frac{dW^{\mu\nu}(x)dW^{\alpha\beta}(y)}{dx^4}\right\rangle \times\right.$$

$$\left.\times\frac{\delta^2\Phi}{\delta R^{\mu\nu}(x)\delta R^{\alpha\beta}(y)}\right) = \int DR\Phi\left(\int\sqrt{-g}dx^0\Lambda...\Lambda dx^3\,\frac{\delta}{\delta R^{\mu\nu}(x)}R^{\mu\nu}(x)f\left[R^{\mu\nu}(x)\right] + \frac{1}{2}\int(-g)dx^0\Lambda...\Lambda dy^3\times\right.$$

$$\left.\times\frac{\left\langle dW^{\mu\nu}(x)dW^{\alpha\beta}(y)\right\rangle}{dx^4}\frac{\delta^2 f\left[R^{\mu\nu}(x)\right]}{\delta R^{\mu\nu}(x)\delta R^{\alpha\beta}(y)}\right) = \int DR\Phi\left[R^{\mu\nu}\right]\int\sqrt{-g}dx^0\Lambda...\Lambda dx^3\left(\frac{\delta}{\delta R^{\mu\nu}(x)}R^{\mu\nu}(x) + \right.$$

$$\left. + \frac{1}{2}Q\frac{\delta^2}{\delta R^{\mu\nu}(x)\delta R_{\mu\nu}(x)}\right)f\left[R^{\mu\nu}(x)\right], \tag{11}$$

in this conclusion averaging is understood in the sense of:

$$\langle ...\rangle = \int DRD(dW)f[R]\rho[dW]..$$

In view of the arbitrariness of the functional Φ, (11) should be functional Fokker-Planck equation (10). "Stationary" solution of (10), which is independent of the choice of a 4-dimensional space-time cross-section in a 5-dimensional extended phase space of the universe, we have found from the condition:

$$\left\{R^{\mu\nu} + \frac{Q}{2}\frac{\delta}{\delta R_{\mu\nu}}\right\}f_0\left[R^{\mu\nu}\right] = 0,$$

which gives:

$$f_0 = C\exp\left\{-\frac{1}{Q}\int\sqrt{-g}dx^0\Lambda...\Lambda dx^3 R^{\mu\nu}(x)R_{\mu\nu}(x)\right\}. \tag{12}$$

It describes the geometry of Minkowski space, distorted "ripples" random fluctuations in the amplitude of the order Q. Fluctuations of this type may be related to fluctuations in the number of gravitons per unit 4-dimensional volume. It was in such a state of initial geometry of space-time according to the cosmological ideas P. Fomin, and E. Trayon [9].

Density functional probability distribution for the curvature of the universe $f\left[R^{\mu\nu}\left(x^{\alpha}\right)\right]$ allows us to calculate the probability W that at the time $t=\frac{x^0}{c}$ the geometry of the universe (her rolled curvature) will look like $R^{\mu\nu}\left(x^k,x^0\right)$, if at the time $t_0=\frac{x_0^0}{c}$ it was the curvature of the form $R^{\mu\nu}\left(x^k,x_0^0\right)$:

$$W=\int_{R^{\mu\nu}\left(x^k,x_0^0\right)}^{R^{\mu\nu}\left(x^k,x^0\right)} D\widetilde{R}^{\mu\nu}\left(x^{\alpha}\right)\exp\left\{-\frac{1}{Q}\int\sqrt{-g}\,dx^0\Lambda...\Lambda dx^3\widetilde{R}^{\mu\nu}\left(x^{\alpha}\right)\widetilde{R}_{\mu\nu}\left(x^{\alpha}\right)\right\}, \tag{13}$$

here k = 1, 2, 3.

Such an approach to stochastic cosmological differs from conventional approaches used in quantum cosmological [10-14] in which the integrals like (13), considered as axioms as the probability amplitude, the wave function of the universe.

However, such approaches do not have explicit physical sense, given that R is not dynamic in the sense of a variable quantum mechanics as immeasurable. In the formula (13) $R^{\mu\nu}\left(x^{\alpha}\right)$ is regarded as a certain elementary random event that occurs with probability $\exp\left\{-\frac{1}{Q}\int\sqrt{-g}\,dx^0\Lambda...\Lambda dx^3\widetilde{R}^{\mu\nu}\widetilde{R}_{\mu\nu}\right\}$, and the expression (13) as the probability of a complex random event, consisting of a set of elementary events. In the same approaches Vilenkin and Hartlle [10, 11, 14] a formula similar to (13) with Q = ħ or 1 only seen as an expression for the probability amplitude, which is not quite clear physical meaning.

2. SELF-ORGANISATION OF FLUCTUATIONS

We now show that, under certain conditions, fluctuations of the geometry of Minkowski space-time can self-organize in a 4-dimensional multiverse Guth-Linde-Vilenkin by Darwinian selection modes [7] To do this, we transform (10) in the non-linear equation, using Klimontovich ideas about the correlation of fluctuations of the Hamiltonian dynamical system with the fluctuations of the probability density function [15]. The role of the Hamiltonian in equation (10) executes the statement:

$$\int\sqrt{-g}\,dx^0\Lambda...\Lambda dx^3\left\{\frac{\delta}{\delta R^{\mu\nu}(x)}R^{\mu\nu}(x)+\frac{Q}{2}\frac{\delta^2}{\delta R^{\mu\nu}(x)\delta R_{\mu\nu}(x)}\right\}\equiv H,$$

$$\frac{d}{dx^4}f\left[R^{\mu\nu}(x)\right]=Hf\left[R^{\mu\nu}(x)\right].$$

Therefore the use of this idea Klimontovich [17] reduces (10) to the form:

$$\frac{d}{dx^4}\left(\langle f\rangle+\delta f\right)=\left(H+\delta H\right)\left(\langle f\rangle+\delta f\right), \tag{14}$$

Averaging this equation, we obtain the Fokker-Planck equation with the collision integral in the general form proposed by Klimontovich [17]:

$$\frac{d\langle f\rangle}{dx^4} = H\langle f\rangle + \langle \delta H \delta f\rangle = \Phi(\langle f\rangle). \tag{15}$$

Here we assume that $\langle \delta H \delta f\rangle$ the functionality of the $\langle f\rangle$.

We now show that, under certain conditions, fluctuations of the geometry of Minkowski space-time (in a sense of x^4) can self-organize in a 4-dimensional multiverse Guth-Vilenkin-Linde [1-3] along the coordinates by Darwinian selection modes (H. Haken) [7].

Let (12) is a stationary solution of equation (15), whereas others find the stationary solutions can be following the ideas of Haken [7] . To do this, we expand the $\langle f\rangle$ solution of (15) with respect to the eigenvectors of the operator $\Phi'[f_0]$, which will assume a self-adjoint, in the neighborhood of the vector f_0:

$$\langle f\rangle = f_0 + \sum_{\gamma} C_\gamma(x^4)\xi_\gamma[R_{\mu\nu}] \text{ and } (16)$$

$$\Phi'(f_0)\xi_\gamma[R_{\mu\nu}] = \alpha_\gamma \xi_\gamma[R_{\mu\nu}].$$

We substitute (16) into the linearized in the neighborhood of equation (15), we obtain:

$$\sum_{\gamma} \frac{d}{dx^4} C_\gamma(x^4)\xi_\gamma = \Phi'(f_0)\sum_{\gamma} C_\gamma(x^4)\xi_\gamma = \sum_{\gamma} \alpha_\gamma C_\gamma \xi_\gamma.$$

Using the orthogonality and normalization of the eigenvectors of a self-adjoint operator

$$\int DR\xi_\alpha[R_{\mu\nu}]\xi_\gamma[R_{\mu\nu}] = \delta_{\alpha\gamma}\ , \tag{17}$$

we obtain an equation for the expansion coefficients (16)

$$\frac{dC_\sigma}{dx^4} = \alpha_\sigma C_\sigma. \tag{18}$$

The decision of which $C_\sigma = C_\sigma^0 \exp[\alpha_\sigma x^4]$ shows that, if anything $\alpha_\sigma \langle 0$, and then $\lim\limits_{x^4 \to +\infty} C_\sigma = 0$ in this part of the 5-dimensional manifold defined by the condition $x^4 \rangle 0$, the expansion (16) leads to the expression $\lim\limits_{x^4 \to +\infty} \langle f\rangle = f_0$, and this part will mainly be a space-time whose metric fluctuates around the Minkowski metric. If $\langle \delta H \delta f\rangle$,however, such that there α_σ exists at least of one of which $\alpha_\mu \rangle 0$, in this case the state of the curvature described by the probability density f_0 becomes unstable x^4 and it is necessary to solve non-linear equation (15). For this, following (H. Haken) [7], consider the decomposition (16) as follows:

$$\langle f\rangle = f_0 + C_\mu(x^4)\xi_\mu[R_{\beta\gamma}] \tag{19}$$

and substitute it into equation (15):

$$\frac{d}{dx^4}C_\mu\left(x^4\right)\xi_\mu=\Phi\left(f_0+C_\mu\left(x^4\right)\xi_\mu\right)$$

or after multiplying by ξ_μ and integration:

$$\frac{d}{dx^4}C_\mu\left(x^4\right)=\int DR\xi_\mu\left[R_{\rho\nu}\right]\Phi\left(f_0\left[R_{\rho\nu}\right]+C_\mu\left(x^4\right)\xi_\mu\left[R_{\rho\nu}\right]\right)=\Psi\left(C_\mu\left(x^4\right)\right). \quad (20)$$

This equation can be easily integrated in quadratures:

$$\int_0^{C_\mu}\frac{dy}{\Psi(y)}=x^4. \quad (21)$$

Its steady-state solution $\widetilde{C}_\mu$ satisfies the equation $\Psi\left(\widetilde{C}_\mu\right)=0$ and describes the probability density function for $R_{\mu\nu}$ fixed values of the form, which describes the geometry of space-time (multiverse) located on the "distance" from the fluctuating Minkowski space-time, where:

$$\int_0^{\widetilde{C}_\mu}\frac{dy}{\Psi(y)}=\widetilde{x}^4. \quad (22)$$

At a certain form of the operator $\Phi'\left(f_0\right)$, this geometry is described by the probability density function $\langle f\rangle_1$:

$$\langle f\rangle_1=f_0+\widetilde{C}_\mu\xi_\mu\left[R_{\rho\nu}\right]=C\exp\left\{-\frac{1}{Q}\int\sqrt{-g}dx^0\Lambda...\Lambda dx^3R^{\alpha\beta}(x)R_{\alpha\beta}(x)\right\}+\widetilde{C}_\mu\xi_\mu\left[R_{\rho\nu}\right]=$$

$$=C_1\exp\left\{-\frac{1}{Q}\int\sqrt{-g}dx_0\Lambda...\Lambda dx^3\left[R^{\alpha\beta}(x)-\chi\left(T_1^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}T_1\right)\right]\left[R_{\alpha\beta}(x)-\chi\left(T_{1\alpha\beta}-\frac{1}{2}g_{\alpha\beta}T_1\right)\right]\right\}. \quad (23)$$

Then for the area of 5D supermultivers the most probable distribution of the geometry $\widetilde{R}_{\alpha\beta}$ is given by the equation: $0=\dfrac{\delta\langle f\rangle_1}{\delta R_{\alpha\beta}(x)}$.

From which it follows that $\widetilde{R}_{\alpha\beta}(x)=\chi\left(T_{1\alpha\beta}-\dfrac{1}{2}g_{\alpha\beta}T_1\right)$, which shows that in this area a 5-dimensional geometry supermultiverse organized in a 4-dimensional multiverse, whose geometry is described by the Einstein-Hilbert equation with a certain energy-momentum tensor $T_1^{\alpha\beta}$.

We assume that in addition to, f_0 and $\langle f_1\rangle$ there are other stationary solutions (15) and the operator $\Phi'\left(\langle f_1\rangle\right)$ has one eigenvalue $\beta_\nu\rangle 0$, then the following (H. Haken), we can find another stationary solution (15) of the form:

$$\langle f\rangle_2 = f_0 + \widetilde{C}_\mu \xi_\mu [R_{\rho\nu}] + \widetilde{C}_\nu \eta_\nu [R_{\rho\nu}] =$$
$$= C_2 \exp\left\{-\frac{1}{Q}\int\sqrt{-g}dx^0\Lambda...\Lambda dx^3\left[R^{\alpha\beta}(x) - \chi\left(T_2^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}\,T_2\right)\right]\left[R_{\alpha\beta}(x) - \chi\left(T_{2\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T_2\right)\right]\right\}. \tag{24}$$

Here η_ν is the eigenvector $\Phi'(\langle f\rangle_1)$ with eigenvalue β_ν.

Similarly, we can break all the 5-dimensional space on a 4-dimensional layers – the space-times, the curvature of which will have a probability density function of the form: $\langle f\rangle_i = C_i$ *

$$\exp\left\{-\frac{1}{Q}\int\sqrt{-g}dx^0\Lambda...\Lambda dx^3\left[R^{\alpha\beta}(x) - \chi_{i-1}\left(T_i^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}\,T_i\right)\right]\left[R_{\alpha\beta}(x) - \chi_{i-1}\left(T_{i\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T_i\right)\right]\right\}$$

and its most likely form will satisfy the equation

$$\frac{\delta\langle f\rangle_i}{\delta R_{\alpha\beta}} = 0 = R^{\alpha\beta}(x) - \chi_{i-1}\left(T_i^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}T_i\right). \tag{25}$$

A place where there is a layer of space with new geometry and its width (to x^4) related to Darwinian selection modes (H. Haken) [7]. For example, when the expansion of the fluctuations f_0 to the eigenvectors of the operator $\Phi'(f_0)$ (Eq. (16)) there is a vector ξ_μ with $\lambda_\mu > 0$, the stationary solution f_0 becomes unstable and the system (15) is transformed into another stationary point $f_0 + \widetilde{C}_\mu\xi_\mu$ as soon as the fluctuations in the expansion of the stationary point appears vector η_ν system becomes unstable and goes into following stationary point $f_0 + \widetilde{C}_\mu\xi_\mu + \widetilde{C}_\nu\eta_\nu$ etc. Therefore, the place and the width of the layers are random. Each 4-dimensional cross-section of 5-dimensional space perpendicular to the $\vec{e}_4$ axis directed along this multiverse Guth-Vilenkin-Linde.

Generally speaking, the energy-momentum tensor $T_i^{\alpha\beta}(x)$ of the multiverse can have whatever kind. However, for us, the most interesting are those multiverse, in which energy-momentum tensor $T_i^{\alpha\beta}(x)$ has the form similar to the energy-momentum tensor of our universe. The equation for the curvature of the space-time is given by the Hilbert-Einstein equations:

$$R_\beta^{\mu\nu} = \chi\left(T_\beta^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T_\beta\right) \tag{26}$$

Here χ – the gravitational constant, $T_\beta^{\mu\nu}$ – the energy-momentum tensor of the scalar field defined for each local universe. This scalar field is a mathematical model of dark energy, therefore, (26) is a good mathematical model of the geometry of our universe, as it other than dark energy (74%), there is dark matter (23%), the nature of which is unknown, and very few ordinary matter (only 4-5%). Relying on the principle of mediocrity

(A. Vilenkin 1995), can be considered the equation (26) as a good mathematical model for the majority of other interesting for us local universes.

3. A SIMPLE EXAMPLE

Choose a metric for the local universe in the form of:

$$g = dt \otimes dt - a^2 \{d\chi \otimes d\chi + \sin^2 \chi (d\theta \otimes d\theta + \sin^2 \theta d\varphi \otimes d\varphi)\}. \tag{27}$$

We assume local universes 3-dimensional hyperspheres variable radius, depending on the "cosmological" time t. Substituting the metric (27) in equation (26), to obtain the tensor components R_{00}:

$$R_{00} = -3\frac{\ddot{a}}{a} = \chi\left(T^{\beta}_{00} - \frac{1}{2} g_{00} T^{\beta}\right) \tag{28}$$

Equation (26) also gives an equation for the scalar field g:

$$\partial^{\alpha}\partial_{\alpha} g + \Gamma^{\mu}_{\mu\nu}\partial^{\nu} g + U'_{\beta}(g) = 0 \tag{29}$$

where $\Gamma^{\mu}_{\mu\nu}$ is collapsed Christoffel symbol, and $U_{\beta}(g)$ is the potential energy of the cosmological constant ("dark energy") g in the β local universe.

In the model, of local universe Linde [16] is expected the field dependence g only on the time t, that for the equations (28) and (29) gives:

$$\begin{cases} \dfrac{\ddot{a}}{a} = -\dfrac{\chi}{3}\{\dot{g}^2 - U_{\beta}(g)\} \\ \ddot{g} + 3\dfrac{\dot{a}}{a}\dot{g} + U'_{\beta}(g) = 0 \end{cases} \tag{30}$$

It is easy to find a particular solution of equation (30):

$$g = g_0 = const\,,\quad U'_{\beta}(g_0) = 0, \tag{31}$$

$$\ddot{a} = \frac{\chi}{3} U_{\beta}(g_0) a\,.$$

If in (31) $U_{\beta}(g_0)\langle 0$ then

$$\ddot{a} = -\frac{\chi}{3}\left|U_{\beta}(g_0)\right| a \tag{32}$$

and the solution is:

$$a = a_2 \sin\left(\sqrt{\frac{\chi}{3}\left|U_{\beta}(g_0)\right|}\, t + \alpha\right) \tag{33}$$

$a_1 = a_2 \sin\alpha -$ the initial radius of the universe, a_2 – maximum radius of the universe, T – the start time of compression of the universe:

$$\sin\left(\sqrt{\frac{\chi}{3}\left|U_{\beta}(g_0)\right|}\, T + \alpha\right) = 1,$$

it is equal to:

$$T = \frac{\frac{\pi}{2} - \alpha}{\sqrt{\frac{\chi}{3}\left|U_\beta(g_0)\right|}}.$$

For this type of universe obeys the law of conservation of energy (the first integral equation (32)) has the form:

$$\ddot{a}\dot{a} = \frac{d}{dt}\left(\frac{\dot{a}^2}{2}\right) = -\frac{\chi}{3}\left|U_\beta(g_0)\right|\frac{d}{dt}\left(\frac{a^2}{2}\right)$$

or

$$\frac{d}{dt}\left(\frac{\dot{a}^2}{2} + \frac{\chi}{3}\left|U_\beta(g_0)\right|\frac{a^2}{2}\right) = 0,$$

$$\frac{\dot{a}^2}{2} + \frac{\chi}{3}\left|U_\beta(g_0)\right|\frac{a^2}{2} = \frac{\dot{a}_1}{2} + \frac{\chi}{3}\left|U_\beta(g_0)\right|\frac{a_1^2}{2} = const,$$

here a_1 , $\dot{a}_1$ are the radius and the rate of expansion of the universe after the beginning of inflation. If in (31) $U_\beta(g_0) \rangle 0$, then

$$\ddot{a} = \frac{\chi}{3}\left|U_\beta(g_0)\right|a \ , \tag{34}$$

and the solution is:

$$a = a_0 \exp\left\{\sqrt{\frac{\chi}{3}\left|U_\beta(g_0)\right|}t\right\}. \tag{35}$$

In the case of such a universe law of conservation of energy (the first integral (34)) is:

$$\ddot{a}\dot{a} = \frac{d}{dt}\left(\frac{\dot{a}^2}{2}\right) = \frac{d}{dt}\left(\frac{\chi}{3}\left|U_\beta(g_0)\right|\frac{a^2}{2}\right)$$

or $$\frac{\dot{a}^2}{2} - \frac{\chi}{3}\left|U_\beta(g_0)\right|\frac{a^2}{2} = \frac{\dot{a}_0^2}{2} - \frac{\chi}{3}\left|U_\beta(g_0)\right|\frac{a_0^2}{2} = const,$$

where a_0 , $\dot{a}_0$ are the radius and the rate of the universe expansion after the inflation start. When the universe has a 0-th energy [16], we get

$$\frac{\dot{a}}{a} = H = \sqrt{\frac{\chi}{3}\left|U_\beta(g_0)\right|} \ , \tag{36}$$

here H is the "constant" of Hubble.

For more details of the cosmological model, we need to know the values a_0 and a_1. To calculate them look at a specific model of inflation at the beginning.

Let the potential energy of the field g is given by Figure 1, then as long as the field oscillates around the value g = 0, then $U_\beta(g)\langle 0$ and the equation (32) – this is the equation of the pendulum. The Lagrangian of this field is equal $L = \frac{\dot{a}^2}{2} - \frac{\chi}{6}\left|U_\beta(0)\right|a^2$.

Indeed $\frac{d}{dt}\frac{\partial}{\partial \dot{a}} L = \ddot{a} = \frac{\partial}{\partial a} L = -\frac{\chi}{3}\left|U_\beta(0)\right|a = \frac{\chi}{3}U_\beta(0)a$

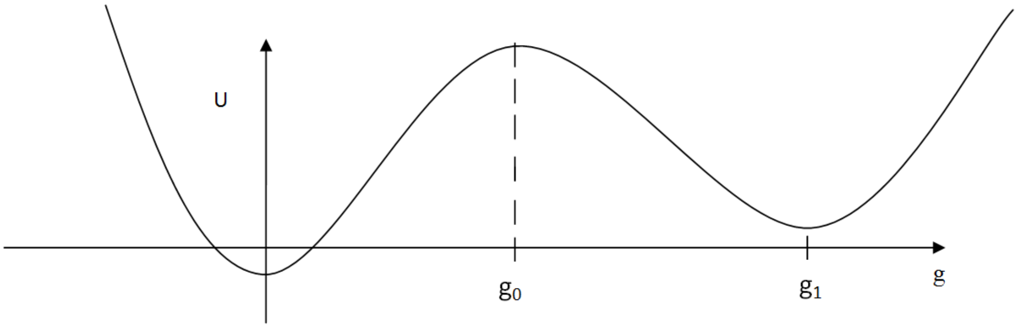

Fig. 1. The potential energy of the field.

It corresponds to the Hamiltonian H:

$$H = p\dot{a} - L = \frac{\dot{a}^2}{2} + \frac{\chi}{6}\left|U_\beta(0)\right|a^2 = \frac{p^2}{2} + \frac{\chi}{6}\left|U_\beta(0)\right|a^2 \tag{37}$$

We assume that in t = 0 and g = 0 the universe was in the ground state of a quantum oscillator with the Hamiltonian (37). Then, the wave function of the universe (at g = 0) was:

$$\psi_0(a) = \left(\frac{\chi}{3}\left|U_\beta(0)\right|\right)^{\frac{1}{8}} \frac{1}{(\pi\hbar)^{\frac{1}{4}}} \exp\left(-\frac{\sqrt{\left|U_\beta(0)\right|\frac{\chi}{3}}}{2\hbar}a^2\right). \tag{38}$$

What gives to a_0 from the formula (35) next value:

$$a_0 = \left(\int_{-\infty}^{\infty} da\left|\psi_0(a)\right|^2 a^2\right)^{\frac{1}{2}}. \tag{39}$$

When the fluctuations will lead the field g at the top of the hill of potential energy U, the neighborhood values g_0, $U_\beta(g_0)$ will be > 0, which would lead to "measure" the radius of the universe and the average value of this measure will be a_0. If the total energy of the emerging universe of dark matter and ordinary matter will exceed the energy inflaton, then the universe will shrink back to the point. If the volume of the universe has arisen is like our universe, to include the energy of the inflaton, that is a field that exceeds the sum of the energies of dark and ordinary matter, the radius of the universe is expanding at an exponential equation (35), where a_0 the initial radius of the universe.

To construct a more accurate cosmological models we must solve the system (30), which is planned in subsequent papers.

CONCLUSION

In this paper, we show that multiverse of Gut-Vilenkin-Linde, may just be the 4-dimensional section, among many other multiverse, 5D supermultiverse. It is shown that the fluctuations of the geometry that arise during the 5th Dimension, "along" which are 4-dimensional sections, "multiverse" of supermultiverse, can self-organize to create a variety of 4-dimensional geometry of the cross-sections.This result was obtained by generalizing the Hilbert-Einstein equations to account for the first stochastic fluctuations of the metric by the stochastic quantization method Haken [7], and then the possibility of self-organizing geometry (on the 5th coordinate). To do this first in a generalized Hilbert-Einstein equations was by Klimontovich [17] a generalized collision integral, which makes non-linear equation, then the equation obtained was analyzed by Haken [7]. Analysis showed that the stationary (in terms of the 5th dimension) correspond to solutions of 4-dimensional geometries multiverse generated inflaton with different structure of the potential energy that corresponds multiverse various laws of nature. The dynamics of the inflationary expansion of various multiverse is also different – it can both fit the model proposed by Guth, Vilenkin and Linde.

Thus, this work provides a new approach to the theory of eternal inflation, showing the mechanisms which can lead to this kind of multiverse.

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Майорова А. М. П'яти-мірна геометрія супермультіверса як результат синергетичної самоорганізації стохастичних флуктуацій геометрії Мінковського / А. М. Майорова, Ю. Д. Мендигулов, Ю. М. Мицай // Вчені записки Таврійського національного університету імені В. И. Вернадського. Серія : Фізико-математичні науки. – 2013. – Т. 26 (65), № 2. – С. 95-108.
Справжня робота є природним синтезом двох сучасних космологічних теорій: п'яти мірних космологічних теорій, які виникли у розвиток єдиної теорії поля запропонованої Т. Калузо, і теорією вічної інфляції Гута-Лінде-Віленкіна.Теорія вічної інфляції не дає відповіді на питання про те, чому «інфлатон» має таку, а не якусь іншу форму його потенційної енергії. Наш підхід заснований на розгляді геометрії як динамічної системи, а також використанні пятимерного простору в якому роль «часу» грає п'ята координата. Дінаміка такої системи описується рівнянням Е. Картана, рішення якого визначає геометрію в нашому 5-вимірному різноманітті. Це рівняння спочатку стохастично квантуєтся по Хакену а потім перетворюється в нелінійне використовуючи ідеї Клімонтовіча. Отримане нелінійне узагальнення рівняння Фоккера-Планка описує самоорганізацію флуктуацій у просторі Мінковського.
Ключові слова: інфлатон, всесвіт, коливання, самоорганізація, рівняння Гільберта-Ейнштейна, енергія-імпульс.

Майорова А. Н. Пяти-мерная геометрия супермультиверса как результат синергетической самоорганизации стохастических флуктуаций геометрии Минковского /А. Н. Майорова, Ю. Д. Мендыгулов, Ю. Н. Мицай // Ученые записки Таврического национального университета имени В. И. Вернадского. Серия : Физико-математические науки. – 2013. – Т. 26 (65), № 2. – С. 95-108.
Настоящая работа является естественным синтезом двух современных космологических теорий: пятимерных космологических теорий, которые возникли в развитие единой теории поля предложенной Т. Калузо, и теорией вечной инфляции Гута-Линде-Виленкина. Теория вечной инфляции не дает ответа на вопрос о том, почему «инфлатон» имеет такую, а не какую-то другую форму его потенциальной энергии. Наш подход основан на рассмотрении геометрии как динамической системы, а также использовании пятимерного пространства, в котором роль «времени» играет пятая координата. Динамика такой системы описывается уравнением Э. Картана, решение которого определяет геометрию в нашем 5-мерном многообразии. Это уравнение сначала стохастически квантуется по Хакену а затем преобразуется в нелинейное используя идеи Климонтовича. Полученное нелинейное обобщение уравнения Фоккера-Планка описывает самоорганизацию флуктуаций в пространстве Минковского.
Ключевые слова: инфлатон, вселенная, колебания, самоорганизация, уравнения Гильберта-Эйнштейна, энергии-импульса.

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