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SPECTRA OF ELEMENTARY EXCITATIONS AND PHASE DIAGRAM OF NON-HEISENBERG SPIN-2 MAGNETIC

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The spectra of elementary excitations have been investigated for spin-2 non-Heisenberg magnetic with the account of all spin invariants. Analysis of the spectra of elementary excitations allowed to construct the phase diagram of the magnet at various relationship between the exchange integrals. In case of single-sublattice magnet, there is equivalence with the phase states and excitation spectra behavior of spin-2 Bose-gas of ultracold atoms.
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INTRODUCTION

Recently, the investigation of magnetically ordered systems with the account of high-order spin invariants has drawn great attention [1-4], because of the fact that such systems are equivalent to the Bose-condensate of "cold" atoms. This condensate can be obtained with the help of various "atom traps" [5, 6]. One can make one or another type of interaction corresponding to spin invariant prevailing by varying trap's parameters. As it was noted in [1], the investigation of such systems can be carried out within the frameworks of the exchange Hamiltonian. Research of the model Hamiltonian with the account of high-order spin invariants [7, 8] allows to find the phase states of the system and also to determine the behavior of excitations spectra near the phase transition lines. Besides, it is possible to determine the relationship between the exchange constants and the parameters of scattering lengths of the corresponding spin systems. It was found during investigations [9, 10] that the increase of the spin of a magnetic ion results in the emergence of the new quantum effects, in particular, to the realization of new nematic phases: tetrahedral and antitetrahedral. It was shown that the geometrical image is a biaxial ellipsoid, while in spin-1 case [11] the geometrical image of the nematic phase is single-axis ellipsoid. It should be also noted that there appears additional parameter in tetrahedral (antitetrahedral) phase – the pseudospin which transforms like the real spin vector at time reflection $t \to -t$. The next stage in the investigation of this system is the investigation of the behavior of the spectra of elementary excitations in the vicinity of the phase transitions lines.

1. PHASE STATES

The aim of the present work is to investigate the phase states and spectra of elementary excitations in the vicinity of the phase transitions lines of the isotropic spin-2 ferromagnetic with the account of the complete set of spin invariants. The Hamiltonian of such a system has the following form:

$$\mathcal{H}=-\frac{1}{2}\sum_{n\neq n'}\left\{J_{nn'}\left(\vec{S}_n\vec{S}_{n'}\right)+K_{nn'}\left(\vec{S}_n\vec{S}_{n'}\right)^2+D_{nn'}\left(\vec{S}_n\vec{S}_{n'}\right)^3+F_{nn'}\left(\vec{S}_n\vec{S}_{n'}\right)^4\right\} \quad (1)$$

where J, K, D, F are the exchange integrals corresponding to various spin invariants. It is supposed that the system considered is at low temperature ($T << T_C$ where T_C is the Curie temperature), as the quantum properties of the system are more evident in this case.

Before we proceed to the investigation of the spectra, we want to remind which phases realize in the system at various relationships between the constants of exchange interactions [10].

1. If the relationship between exchange integrals is such that $J_0 > K_0, D_0, F_0$, then the wave-function of the ground state is given by $\left|\psi_{gr.st}\right\rangle=\left|2\right\rangle$. Therefore, the averages on this state are $\left\langle S^z\right\rangle=2, \left\langle\left(S^z\right)^2\right\rangle=4, \left\langle\left(S^x\right)^2\right\rangle=\left\langle\left(S^y\right)^2\right\rangle=1$. This state is ferromagnetic (*FM*).

2. At $K_0 > J_0, D_0, F_0$, the wave-function of the ground state is $\left|\psi_{gr.st.}\right\rangle=\frac{1}{\sqrt{2}}\cos\beta\left(\left|2\right\rangle+\left|-2\right\rangle\right)+\sin\beta\left|0\right\rangle$. This state is characterized by the quadrupolar order parameters: $q_2^0=3\left\langle(S^z)^2\right\rangle-6=6\cos 2\beta$, $q_2^2=\left\langle(S^x)^2\right\rangle-\left\langle(S^y)^2\right\rangle=2\sqrt{3}\sin 2\beta$, because $\left\langle S\right\rangle=0$. This spin state is "spin nematic" and will be denoted as *N*.

3. And there is one more case: $D_0 > K_0, F_0$. In this case, the wave-function of the ground state looks like $\left|\psi_{gr.st.}\right\rangle=\frac{1}{\sqrt{3}}\left(\left|2\right\rangle+\sqrt{2}\left|-1\right\rangle\right)$. This phase is characterized with order parameters of higher order in spin operators S^i: $q_3^0=10$, $q_3^3=\frac{1}{2}\left(\left\langle\left(S^+\right)^3\right\rangle+\left\langle\left(S^-\right)^3\right\rangle\right)=4\sqrt{2}$, because $\left\langle S^z\right\rangle=0$, $q_2^0=q_2^2=0$. This phase state will be denoted as *TQ*-state.

In case of negative constant of Heisenberg exchange interaction $J_0 < 0$, the two-sublattice magnetically ordered structures realize in the system:

4. Antiferromagnetic (*AFM*) state. In this state, only axial moments are differ from zero: $\left\langle S^z\right\rangle=2$ $q_2^0=6$, $q_3^0=6$, $q_4^0=12$. The order parameters of the second sublattice are $\left\langle S^z\right\rangle=-2$, $q_2^0=6$, $q_3^0=-6$, $q_4^0=12$.

5. And finally, the *ATQ*-antitetrahedral phase, characterized with tensor component of the higher order: $q_3^0=10$, $q_3^3=4\sqrt{2}$, $q_4^0=-28$, $q_4^3=2\sqrt{2}$ in the first sublattice, and $q_3^0=-10$, $q_3^3=-4\sqrt{2}$, $q_4^0=-28$, $q_4^3=2\sqrt{2}$ in the second sublattice.

2. SPECTRA OF ELEMENTARY EXCITATIONS OF SINGLE-SUBLATTICE NON-HEISENBERG SPIN-2 MAGNETIC

The spectra of elementary excitations are determined by the poles of the Green function [1]:

$$G^{\alpha\alpha'}(n,\tau;n',\tau') = -\left\langle \hat{T}\tilde{X}_n^{\alpha}(\tau)\tilde{X}_{n'}^{\alpha'}(\tau')\right\rangle, \tag{2}$$

where $\tilde{X}_n^{\alpha}(\tau) = \exp(\mathcal{H}\tau)X_n^{\alpha}\exp(-\mathcal{H}\tau)$ are the Hubbard operators in the Heisenberg representation; $\hat{T}$ is the Wick operator; $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{int}}$ [12, 13]. The further evaluations will be carried out within the mean-field approximations; therefore we can restrict ourselves with the dynamic part of the exchange Hamiltonian which can be presented as follows:

$$H_{\text{int}} = -\frac{1}{2}\sum_{n\neq n'}\sum_{\alpha,\beta}\left\{\mathbf{c}(\alpha)\hat{A}_{nn'}\,\mathbf{c}(\beta)\right\}\Delta X_n^{\alpha}\cdot\Delta X_{n'}^{\beta}, \tag{3}$$

where $\Delta X^{\alpha} = X^{\alpha} - \left\langle X^{\alpha}\right\rangle_0$, and the components of the 24-dimensional vector $\mathbf{c}(\alpha)$ are determined from the relationship between the spin and tensor operators with the Hubbard operators; $\hat{A}_{nn'}$ matrix is given by

$$\hat{A}_{nn'} = \frac{a_{nn'}^{(1)}}{2}\left(2\hat{E}\oplus\hat{I}\right)\oplus\frac{a_{nn'}^{(2)}}{2}\left(6\hat{E}\oplus\hat{I}\oplus\hat{I}\right)\oplus\frac{a_{nn'}^{(3)}}{160}\left(16\hat{E}\oplus 3\hat{I}\oplus 30\hat{I}\oplus 20\hat{I}\right)\oplus$$

$$\oplus\frac{a_{nn'}^{(4)}}{560}\left(2\hat{E}\oplus 10\hat{I}\oplus 5\hat{I}\oplus 70\hat{I}\oplus 35\hat{I}\right),$$

where $\hat{E}$ is unit matrix, $\hat{I} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

The derivation of the dispersion equation determining the spectra of elementary excitations is given in Refs. [12, 13]. The equation is valid at arbitrary spin value, arbitrary symmetry of the single-ion anisotropy, and arbitrary temperature (except the region of fluctuation). The dispersion equation can be presented in the following form:

$$\det\left\|1 + x_{ij}\right\| = 0,\ i,j = 1,..,24. \tag{4}$$

Solutions of equation (4) determine magnon spectra in different phases.

Let us proceed with the analysis of the spectra of elementary excitations for each phase.

1. Using the explicit form of the order parameters in the *FM*-phase, we can obtain the magnon spectra in the long wave-length limit (at $k \to 0$):

$$\varepsilon_1(k) = 4\left(\tilde{J}_0 + 9\delta k^2\right); \tag{5}$$

$$\varepsilon_2(k) = 4\left[2\left(\tilde{J}_0 + 3\tilde{K}_0\right) + 3(\lambda + \gamma + 13\delta)k^2\right]; \tag{6}$$

$$\varepsilon_3(k) = 2(\alpha + 4\lambda + 16\gamma + 64\delta)k^2; \tag{7}$$

$$\varepsilon_4(k) = 6\left[2\left(\tilde{J}_0 + 3\tilde{K}_0\right) + 3(\gamma - 4\delta)k^2\right], \tag{8}$$

where $\tilde{J}_0 = 2J_0 - K_0 + 41D_0 - 79F_0$ and $\tilde{K}_0 = K_0 - 5D_0 + 43F_0$; $J_0 - J_k = \alpha k^2$,

$K_0 - K_k = \lambda k^2,\ D_0 - D_k = \gamma k^2, F_0 - F_k = \delta k^2$.

As it is seen from Eq. (5) and (8), the magnon spectra soften at $\tilde{J}_0 = 0$ and $\tilde{J}_0 + 3\tilde{K}_0 = 0$, i.e., the *FM*-phase becomes unstable. Thus, the phase transition from the *FM*-phase occurs along the magnon branches $\varepsilon_1(k)$ and $\varepsilon_4(k)$. It should be noted that nevertheless the gap in the spectrum $\varepsilon_2(k)$ has the same form as in the spectrum $\varepsilon_4(k)$, the phase transition occurs along the branch (8), as the magnon "velocity" in this branch is higher, than in the branch (6).

2. Now consider the spectra of elementary excitations in the *N*-phase in the long wave-length limit (with the account of the notations, introduced above):

$$\varepsilon_1^2(k) = 48\left[(\lambda - 2\gamma + 28\delta)k^2 \sin^2\beta + 3\cdot\delta k^2 \cos^2\beta\right]\times \times\left[3\tilde{K}_0 \sin^2\beta - \tilde{J}_0 \cos^2\beta\right]; \tag{9}$$

$$\varepsilon_2^2(k) = 144\tilde{K}_0(\lambda - 2\gamma + 28\delta)k^2; \tag{10}$$

$$\varepsilon_{3,4}^2(k) = 12\left[(\lambda - 2\gamma + 31\delta)k^2 + (\lambda - 2\gamma + 25\delta)k^2\cos(2\beta \pm \pi/3)\right]\times \times\left[3\tilde{K}_0 - \tilde{J}_0 + (3\tilde{K}_0 + \tilde{J}_0)\cos(2\beta \pm \pi/3)\right] \tag{11}$$

It is seen that the magnon spectra (9) – (11) in the *N*-phase are linear in the wave-vector far away from the stability points. The branch $\varepsilon_1(k)$ softens at line $\tilde{J}_0 = 0$ when parameter $\beta = 0$ (parameter β is re-determined exactly), and takes the form $\varepsilon_1(k) = 36\delta k^2$. Consequently, the line $\tilde{J}_0 = 0$ is the line of the phase transition $N - FM$ phase, at this, the spin nematic tends to single-axis nematic near this line. Besides, the magnon spectrum (9) becomes unstable at $\tilde{K}_0 = 0$, and at $\beta = \frac{\pi}{2}$ takes the form: $\varepsilon_1(k) = 36(\gamma - 5\delta)k^2$, and the order parameters of the *N*-phase tend to the parameters of the "flat" nematic near this line. It should be noted that the spectrum (10) also loses stability at line $\tilde{K}_0 = 0$, and is proportional to k squared: $\varepsilon_2(k) = 36(\gamma - 5\delta)k^2$. Besides, the line of excitations (11) is degenerated which is related with the degeneracy of the energy levels of a magnetic ion $E_1 = E_{-1}$ in the *N*-phase at $\beta = 0,$ and $\beta = \pi/2$.

3. Now consider the spectra of elementary excitations in the *TQ*-phase. As it was mentioned above, the three-fold degeneracy of the excited energy levels of a magnetic ion is observed in this phase which results in coincidence of three branches of elementary excitations. In the long wave-length limit these spectra have the following form:

$$\varepsilon_{1,2,3}^2(k) = 144(\tilde{J}_0 + 3\tilde{K}_0)(\gamma - 4\delta)k^2; \tag{12}$$

$$\varepsilon_4^2(k) = 144\left(-\tilde{K}_0 + (\lambda - 2\gamma + 28\delta)k^2\right)^2. \tag{13}$$

As it follows from Eq. (12), the spectra $\varepsilon_{1,2,3}(k)$ are linear in the wave-vector k far from the line $\tilde{J}_0 + 3\tilde{K}_0 = 0$, but are square in k near the line $\tilde{J}_0 + 3\tilde{K}_0 = 0$:

$$\varepsilon_{1,2,3}(k) = 18(\gamma - 4\delta)k^2,$$

i.e., the magnon spectra soften on this line, and the line $\tilde{J}_0 + 3\tilde{K}_0 = 0$ is the line of the phase transition *TQ* – *FM* phase. The spectrum (13) is unstable at the line $\tilde{K}_0 = 0$ and becomes square in k:

$$\varepsilon_2(k) = 36(\gamma - 5\delta)k^2.$$

Consequently, the line $\tilde{K}_0 = 0$ is the line of the phase transition *TQ*- *N* - phase.

Thus, the analysis of the magnon spectra allows to construct the phase diagram m of the system considered (see Fig. 1). The coincidence of the spectra at the lines of the phase transitions testifies that these phase transitions are of the second kind. It should be noted that this phase diagram completely coincides with the phase diagram, obtained for ultracold neutral atoms with $S = 2$ [5, 6].

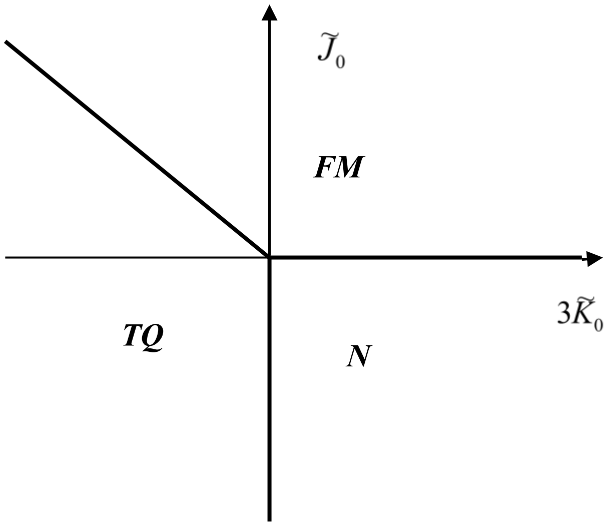

Fig. 1. Phase diagram of non-Heisenberg spin-2 ferromagnetic on the $(\tilde{J}_0, \tilde{K}_0)$-plane.

3. SPECTRA OF ELEMENTARY EXCITATIONS OF TWO-SUBLATTICE NON-HEISENBERG SPIN-2 MAGNETIC

As before, the spectra of elementary excitations are determined by the poles of the Green function. Using order parameters, determined above, we can obtain the spectra of elementary excitations in the corresponding phase states.

1. *AFM*-phase. The magnon spectra in the *AFM*-phase can be presented as the difference of two squares, consequently, their behavior in the center of the Brillouin zone (k = 0) and on the boundary ($k = \pi$), are almost equivalent. The magnon spectra in the *AFM*-phase in the long wave-length limit ($k \to 0$) look like:

$$\varepsilon_1^2(k) = 16\left(\tilde{J}_0' - 18F_0 + 9\delta k^2\right)\left(\tilde{J}_0' - 9\delta k^2\right); \tag{14}$$

$$\varepsilon_2^2(k) = 4\left[\tilde{J}_0' - \tilde{K}_0' - 3(\lambda - 5\gamma + 43\delta)k^2\right]\left[\tilde{J}_0' - 3\tilde{K}_0' - 6\left(K_0 - 5D_0 + 43F_0\right)\right]; \tag{15}$$

$$\varepsilon_3^2(k) = 4\left[\tilde{J}_0' - 3\tilde{K}_0' - 6\left(K_0 - 5D_0 + 43F_0\right)\right](\alpha - 5\lambda + 34\gamma - 179\delta)k^2; \tag{16}$$

$$\varepsilon_4^2(k) = 9\left(\tilde{J}_0' - 3\tilde{K}_0' + 6(\gamma - 6\delta)k^2\right)\left(\tilde{J}_0' - 3\tilde{K}_0' + 12\left(D_0 - 6F_0\right)\right). \tag{17}$$

The spectrum (14) softens at the line $\tilde{J}_0' = 2J_0 - K_0 + 41D_0 - 61F_0 = 0$. This line describes the phase transition from the *AFM*-phase into the *N*-state.

The energy gaps in spectra (15) and (17) decrease while approaching to the line of the phase transition into the *ATQ*-phase. Both branches are unstable at the line of the phase transition the *AFM-ATQ* phase $\tilde{J}_0' - 3\tilde{K}_0' = 2J_0 - 4K_0 + 38D_0 - 100F_0 = 0$; however, the velocities of the "spin" waves are different: the phase transition occurs along $\varepsilon_4(k)$, because its velocity coincides with the velocity of the wave in the *ATQ*-phase at line $\tilde{J}_0' - 3\tilde{K}_0' = 0$.

2. Consider the spectra of elementary excitations in the *N*-phase. In the long wave-length limit at $k \to 0$:

$$\varepsilon_1^2(k) = 98\left[(K_0 - 2D_0 + 28F_0)\sin^2\beta + 3F_0\cos^2\beta\right]\times\left[3\tilde{K}_0'\sin^2\beta + \right.$$
$$\left. + \tilde{J}_0'\cos^2\beta - (2\alpha - \lambda + 41\gamma - 70\delta)k^2\cos^2\beta - 9(\gamma - 5\delta)k^2\sin^2\beta\right]. \quad (18)$$

$$\varepsilon_2^2(k) = 144(K_0 - 5D_0 + 43F_0)(\lambda - 2\gamma + 28\delta)k^2; \quad (19)$$

$$\varepsilon_{3,4}^2(k) = 48\left[K_0 - 2D_0 + 31F_0 + (K_0 - 2D_0 + 25F_0)\cos\left(2\beta \pm \frac{\pi}{3}\right) + \alpha' k^2\right]\times$$
$$\times\left[J_0 + K_0 + 22D_0 - 11F_0 - (J_0 - 2K_0 + 19D_0 - 50F_0)\cos\left(2\beta \pm \frac{\pi}{3}\right) + \alpha'' k^2\right], \quad (20)$$

where α' and α'' are the combinations of $\alpha, \lambda, \gamma, \delta$.

As it is seen from Eq. (18), the spectrum softens at the line of the phase transition into the *AFM*-phase (at $\beta = 0$): $\tilde{J}_0' = 2J_0 - K_0 + 41D_0 - 61F_0 = 0$. On the other hand, this spectrum also softens at the line of the phase transition into the *ATQ*-phase (at $\beta = \pi/2$): $\tilde{K}_0' = K_0 + D_0 + 13F_0 = 0$. The branches $\varepsilon_2(k) - \varepsilon_4(k)$, as it easy to notice, do not soften in the vicinity of the phase transitions lines $\tilde{J}_0' = 2J_0 - K_0 + 41D_0 - 61F_0 = 0$ and $\tilde{K}_0' = K_0 + D_0 + 13F_0 = 0$.

3. Now consider the spectra of elementary excitations in the *ATQ*-phase. There observed a three-fold degeneracy of the excited energy levels of a magnetic ion $E_1 = E_{-1} = E_0$ which results in coincidence of three branches of elementary excitations $\varepsilon_1(k) = \varepsilon_2(k) = \varepsilon_3(k)$. In the center of Brillouin zone ($k \to 0$), the spectra are given by

$$\varepsilon_{1,2,3}^2(k) = 72\left[-\tilde{J}_0' + 3\tilde{K}_0' + (2\alpha - 4\beta + 47\gamma - 154\delta)k^2\right](D_0 - 6F_0); \quad (21)$$

$$\varepsilon_4^2(k) = 144\left[\tilde{K}_0' - (\beta - 2\gamma + 28\delta)k^2\right]\left[6(D_0 - 5F_0) - \tilde{K}_0'\right]. \quad (22)$$

The gap in spectrum $\varepsilon_{1,2,3}(k)$ vanishes at the line of the phase transition *ATQ-AFM* phase $\tilde{J}_0' - 3\tilde{K}_0' = 2J_0 - 4K_0 + 38D_0 - 100F_0 = 0$, and the spectrum becomes linear in *k*. The spectrum $\varepsilon_4(k)$ becomes unstable at the line of the phase transition *ATQ-N* phase $\tilde{K}_0' = K_0 + D_0 + 13F_0 = 0$, and becomes linear in *k*. At the boundary of the Brillouin zone ($k \to \pi$), the spectra look like:

$$\varepsilon_{1,2,3}^2(k) = 36\left[\tilde{J}_0' - 3\tilde{K}_0' + 18(D_0 - 6F_0) - (2\alpha - 4\beta + 47\gamma - 154\delta)k^2\right](\gamma - 6\delta)k^2;$$

$$\varepsilon_4^2(k) = 144\left[\tilde{K}_0' - (\beta - 2\gamma + 28\delta)k^2\right] \times \left[(\beta - 2\gamma + 28\delta)k^2 - \tilde{K}_0' + 6(D_0 - 5F_0)\right].$$

Spectrum $\varepsilon_{1,2,3}(k)$ is linear in the wave-vector *k*; however, it is stable at the phase boundaries. Behavior of $\varepsilon_4(k)$ at the boundary of the zone is equivalent to the behavior in the center of the zone. Analysis of the spectra if elementary excitations and of the free energy density allows to construct the phase diagram of the two-sublattice non-Heisenberg magnetic. This phase diagram on the ($\tilde{J}', \tilde{K}'$)-plane is given in Fig. 2.

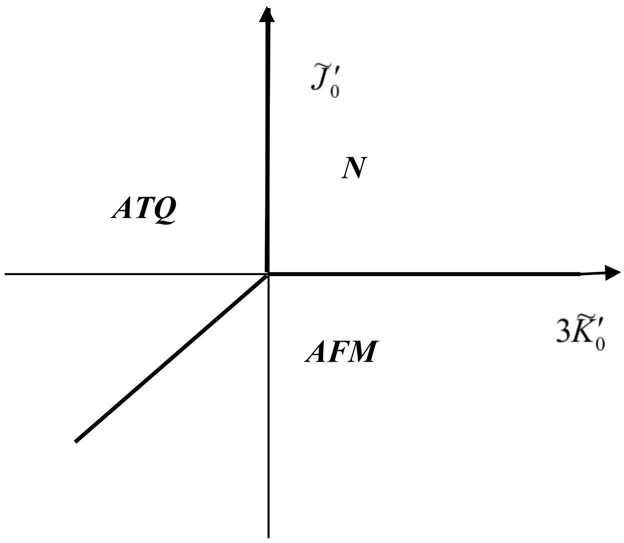

Fig. 2. Phase diagram of two-sublattice non-Heisenberg spin-2 magnetic on the $(\tilde{J}_0', \tilde{K}_0')$-plane.

CONCLUSIONS

The carried out investigations of the spin-2 non-Heisenberg magnetic allow to stay that the account of high-order spin invariants is essential and leads to the realization of the magnetically ordered states with more complex structure, than ferro- or antiferromagnetic. The nematic phase together with the tetrahedral and the antitetrahedral phases belongs to such more complex states. These phases are characterized with that their magnetization (per site) equals zero, while the states, realized in them, are magnetically ordered, and the order parameters are the components of the tensor of quadrupolar moments. The states with zero magnetization per site, but with finite multipole order parameters are purely quantum effect [14]. Nevertheless the fact that magnetization equals zero in these phases, these phases are different, because they have different ground states, different topology in the spin space, and, consequently, different symmetry. The key feature of these phases for $S = 2$ is their more complex structure (the geometrical images in the spin space), in comparison with the structure of the nematic phases, investigated previously for $S = 1$ and $S = 3/2$ [9-11]. Thus, the nematic phase is a single-axis ellipsoid in spin-1 magnetic; while its geometrical image in the case considered is "goffered" bi-axial ellipsoid which loses its "goffering" and becomes single-axis only at the lines of the phase transitions. Besides, the antinamatic phase is absent in spin-2 magnetic, while it is observed in magnetic with the spin of a magnetic ion $S = 3/2$. However, the tetrahedral/antitetrahedral phase can realize in the system under consideration which, in some way, is analogues of the antinematic phase. However, the tetrahedral/antitetrahedral phase has more complex geometrical structure (in the spin space) in comparison with the antinematic phase in spin-3/2 magnetic; however, similar to the magnetic with $S = 3/2$ the tetrahedral/antitetrahedral phase has additional order parameter – the pseudospin $\vec{\sigma}$ which is described by non-zero averages from expressions cubic in spin operators. It should be noted that the appearance of the states with the pseudospin order parameter is possible only in non-Heisenberg magnets with $S > 1$, because this parameter describes with non-zero averages from expressions cubic in spin operators.

Thus, the mean-field analysis of the non-Heisenberg magnetic with spin-2 allowed us to describe both, the dynamic, and the static properties of the system, to reveal formation peculiarities of the phases with multipole order parameters, and to construct the phase diagram of the system.

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Космачов О. О. Спектри елементарних збуджень негейзенберговского магнетика зі спіном S = 2 / О. О. Космачов // Вчені записки Таврійського національного університету імені В. І. Вернадського. Серія : Фізико-математичні науки. – 2013. – Т. 26 (65), № 2. – С. 60-68.
У роботі досліджено спектри елементарних збуджень негейзенберговського магнетика зі спіном магнітного іона 2, при обліку всіх спінових інваріантів. Аналіз спектрів елементарних збуджень дозволив побудувати фазову діаграму магнетика при різних співвідношеннях обмінних інтегралів. У разі однопідграткового магнетика є відповідність фазових станів і поведінки спектрів збудження бозе-газу ультрахолодних атомів з S=2.
Ключові слова: негейзенберговський магнетик, фазові переходи, оператори Хабарда, нематична фаза; тетраедрична фаза.

Космачев О. А. Спектры элементарных возбуждений негейзенберговского магнетика со спином S = 2 / О. А. Космачев // Ученые записки Таврического национального университета имени В. И. Вернадского. Серия : Физико-математические науки. – 2013. – Т. 26 (65), № 2. – С. 60-68.
В работе исследованы спектры элементарных возбуждений негейзенберговского магнетика со спином магнитного иона 2, при учете всех спиновых инвариантов. Анализ спектров элементарных возбуждений позволил построить фазовую диаграмму магнетика при различных соотношениях обменных интегралов. В случае одноподрешеточного магнетика имеется соответствие фазовых состояний и поведения спектров возбуждения бозе-газа ультрахолодных атомов с *S*=2.
Ключевые слова: негейзенберговский магнетик; фазовые переходы; операторы Хаббарда; нематическая фаза; тетраэдрическая фаза.

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