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DISTRIBUTION OF THE ANTIFERROMAGNETIC VECTOR FOR A PERIODIC SYSTEM OF REMOTE CIRCULAR ANTIDOTS AND A COUPLE OF CIRCULAR ANTIDOTS IN AN ANTIFERROMAGNETIC FILM

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The paper is dedicated to the theoretical investigation of the antiferromagnetic vector distribution in a film composed of an isotropic or uniaxial antiferromagnet in the presence of circular antidots. A solution of the Landau-Lifshitz equation is obtained for the antiferromagnetic vector in such antidot system. The antiferromagnetic vector distribution is obtained for a periodic system of remote antidots in an isotropic antiferromagnet and for a couple of antidots (not remote, in general) in an isotropic antiferromagnet, "easy axis" antiferromagnet and "easy plane" antiferromagnet with various boundary conditions.
Keywords: antiferromagnet, magnetic thin film, magnetic antidot, antiferromagnetic vector.

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INTRODUCTION

It is known that magnetic nanostructures of different configurations – magnetic quantum dots [1], magnetic thin films [2], magnetic nanospheres [3], nanowires [4] and nanotubes [5], magnetophotonic crystals [6] and other nanostructures – are investigated intensively in recent years. These nanostructures have found a multitude of technical applications – in information storage and transmission devices [7], in magnetic resonance imaging [8], for magnetic refrigeration [9] and so on. In particular, magnetic quantum dots and their systems as well as magnetic antidots [10] (holes in thin films) and their systems are perspective for various technical applications.

Systems of ferromagnetic [11, 12] and antiferromagnetic [13, 14] dots are studied intensively in recent years. However, systems of ferromagnetic [15, 16] and especially antiferromagnetic [17] antidots are poorly researched at the moment, and known studies in this area are devoted mainly to exchange bias in antiferromagnetic antidot arrays. However, antiferromagnetic antidot systems are promising for a variety of technical applications – in information storage devices [18], in magnon waveguides [19], as a basis for magnetic metamaterials [20], as two-dimensional magnonic crystals [21] and so on. Thus, magnetic properties of antiferromagnetic antidots and their systems are an actual topic of research.

We consider a system of circular antidots in an antiferromagnetic film. We obtain a distribution of the antiferromagnetic vector for the following antidot systems (with various boundary conditions): a couple of circular antidots in an isotropic antiferromagnet, a couple of circular antidots in an uniaxial antiferromagnet and a periodic remote antidot system in an isotropic antiferromagnet.

1. SETTING OF THE PROBLEM

Let us consider a two-sublattice antiferromagnetic film with a thickness d and direct Oz axis in the normal to the film direction. Let us also consider a system of circular antidots in this film, with radiuses R_i and in-plane radius-vectors of the antidots axes $\{\boldsymbol{r}_{0i}\}$ (see Fig. 1). We assume that the magnetization density of the antiferromagnet sublattices ($\boldsymbol{M_1}$ and $\boldsymbol{M_2}$, respectively) are equal in magnitude and opposite in direction, so that $\boldsymbol{M_1} = -\boldsymbol{M_2}$, and are constant in magnitude, so that $|\boldsymbol{M_1}| = |\boldsymbol{M_2}| = M_0$, $M_0 = \text{const}$, everywhere in the film. Thus, the total magnetization vector $\boldsymbol{M} = 0$, and the antiferromagnetic vector is also constant in magnitude: $|\boldsymbol{L}| = L_0 = \text{const}$. We also assume that the film antiferromagnet is characterized by the following parameters: uniaxial anisotropy constants β_1 and β_2, non-uniform exchange constants α_1 and α_2 (where $\alpha_1 > 0$), a uniform exchange constant A.

The goal of this work is to find a distribution of the antiferromagnetic vector in the above-described antiferromagnetic film for a periodic remote antidot system and a couple of antidots (in general, not a remote couple) with various boundary conditions.

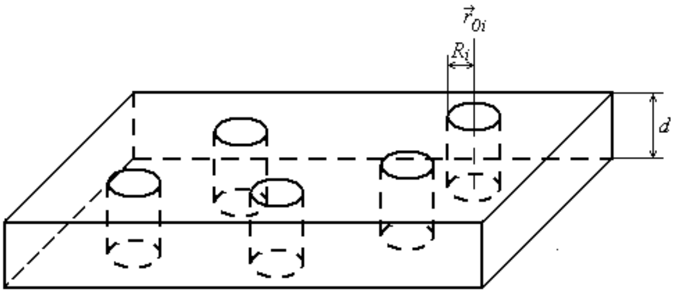

Fig. 1. Antiferromagnetic film investigated in the paper.

2. SOLUTION OF THE LANDAU-LIFSHITZ EQUATION. GENERAL FORM OF THE ANTIFERROMAGNETIC VECTOR DISTRIBUTION

Let us write down the Landau-Lifshitz equation for a static distribution of the antiferromagnetic vector **L** in the antiferromagnetic film that we consider in spherical coordinates (r,θ,φ). If we denote the azimuthal and polar angles of the vector **L** as φ_L and θ_L, respectively, the equation can be written in the following form [22, 23]:

$$\begin{cases} c_0^2 div\left(\sin^2\theta_L \nabla\varphi_L\right) = 0 \\ c_0^2\Delta\theta_L + \left((gH_0)^2 - c_0^2\Delta\varphi_L - \omega_0^2\right)\sin\theta_L\cos\theta_L = 0 \end{cases}, \tag{1}$$

here $\boldsymbol{H_0}$ is the external magnetic field, $c_0 = \dfrac{4\mu_0 M_0}{\hbar}\sqrt{A\alpha_1}$, $\omega_0 = \dfrac{4\mu_0 M_0}{\hbar}\sqrt{A\beta_1}$. In the absence of the external field ($\boldsymbol{H_0} = 0$) the equation (1) – after dividing on the value ω_0^2 – becomes similar to the static equation for the magnetization in an uniaxial ferromagnet with the exchange constant α_1 and the anisotropy constant β_1. This allows us to use the solutions of this equation are given in [23]. Let us select the following solution:

$$\begin{cases} tg\left(\dfrac{\theta_L}{2}\right)=\dfrac{b}{dn\left(c\sqrt{|C_1|}P(X,Y,Z),k_1\right)} \\ \varphi_L=Q(X,Y,Z) \end{cases}, \tag{2}$$

here $X=x/l_0$, $Y=y/l_0$, $Z=z/l_0$; value $l_0=\sqrt{\alpha_1/|\beta_1|}$ for $\beta_1\neq 0$, $l_0=1$ for $\beta_1=0$; C_1 is a constant that lies in the range $-1/4<C_1<0$ for this solution, and the values

$$c=\sqrt{\frac{1+2C_1+\sqrt{1+4C_1}}{2|C_1|}},\ b=\sqrt{\frac{1+2C_1-\sqrt{1+4C_1}}{2|C_1|}},k_1=\sqrt{\frac{2\sqrt{1+4C_1}}{1+2C_1+\sqrt{1+4C_1}}}, \tag{3}$$

$0<k_1\leq 1$ is the modulus of the Jacobi elliptic function. The functions P and Q for the antidot system described in the “Setting of the problem” section can be written down in the following form:

$$\begin{cases} P=\dfrac{\pm z}{l_0}\Theta(\beta_1)+\sum\limits_i n_i\ln\left(\dfrac{|\boldsymbol{r}-\boldsymbol{r}_{0i}|}{l_0}\right)+C_2 \\ Q=\dfrac{\pm z}{l_0}\Theta(-\beta_1)+\sum\limits_i \alpha_i n_i+C_3 \end{cases}, \tag{4}$$

here $\boldsymbol{r}$ is the radius vector of a point in a plane, $\boldsymbol{r}_{0i}=(x_{0i},y_{0i})$ are in-plane radius vectors of the antidots centers, n_i are arbitrary integers, C_2, C_3 are constants, α_i is the azimuthal angle relative to the point $\boldsymbol{r}_{0i}$ (so that $\alpha_i=arctg\left(\dfrac{y-y_{0i}}{x-x_{0i}}\right)$) and the function

$$\Theta(\xi)=\begin{cases}0, & \xi\leq 0\\ 1, & \xi>0\end{cases}. \tag{5}$$

Let us consider specific antidots configurations and find forms of the solution (4) for these configurations with specific boundary conditions.

3. COUPLE OF ANTIDOTS IN AN ISOTROPIC ANTIFERROMAGNET

Let us select the functions P and Q in the following form:

$$\begin{cases} P=n_1\ln\dfrac{|\boldsymbol{r}|}{l_0}+n_2\ln\left(\dfrac{|\boldsymbol{r}-\boldsymbol{r}_0|}{l_0}\right)+C_2 \\ Q=n_1\varphi+n_2\alpha+C_3 \end{cases}, \tag{6}$$

where α is an azimuthal angle relative to the point $\boldsymbol{r}_0$. Such distribution corresponds, in particular, to a couple of circular antidots in an isotropic antiferromagnet with their centers in the origin of coordinates and in the point $\boldsymbol{r}_0$. We consider the couple of antidots of the same radii R; for the sake of convenience we choose the coordinate system so that the axis of the antidot 2 lies on the semiaxis $x > 0$, so that $\boldsymbol{r}_0 = \begin{pmatrix} d \\ 0 \end{pmatrix}$, $d > 0$.

Let us consider the case when these antidots are not remote from one another, so that, in general, the condition $|\boldsymbol{r}_0| >> R$ is not satisfied. Then, the functions P and Q have the following form on the antidots surfaces:

$$\begin{cases} P = n_1 \ln \dfrac{R}{l_0} + n_2 \ln\left(\dfrac{d}{l_0} \sqrt{1 + \dfrac{R^2}{d^2} - 2\dfrac{R}{d}\cos\varphi} \right) + C_2 \\ Q = n_1\varphi + n_2\left(\pi - arctg\, \dfrac{\sin\varphi}{d/R - \cos\varphi} \right) + C_3 \end{cases} \tag{7}$$

on the surface of the antidot 1, and

$$\begin{cases} P = n_1 \ln\left(\dfrac{d}{l_0} \sqrt{1 + \dfrac{R^2}{d^2} + 2\dfrac{R}{d}\cos\alpha} \right) + n_2 \ln\left(\dfrac{R}{l_0} \right) + C_2 \\ Q = n_1 arctg\left(\dfrac{\sin\alpha}{d/R + \cos\alpha} \right) + n_2\alpha + C_3 \end{cases} \tag{8}$$

on the surface of the antidot 2.

As we can see from (7) and (8), a simple and convenient choice of boundary conditions – constant boundary conditions on antidots surfaces – cannot be made in this case because the functions P and Q depend on the local azimuthal angle on the surface of each antidot. However, we can set boundary conditions in some points of the antidots surface, for example, on the intervals $\varphi = 0$ (for the first antidot) or $\alpha = 0$ (for the second antidot). In addition, we can set boundary conditions on some other interval of the antiferromagnetic film, for example, in the point $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$. Let us consider various choices of the boundary conditions.

Boundary conditions 1.

On the surface of the antidot 2 $\theta_L = \pi/2$, $\varphi_L = \varphi_0$ for $\alpha = 0$, on the surface of the antidot 1 $\theta_L = \pi/2$ for $\varphi = 0$.

Substituting these boundary conditions into the solution (2), (6), we obtain

$$\begin{cases} tg\left(\dfrac{\theta_L}{2}\right) = \dfrac{b}{dn\left(c\sqrt{|C_1|}\left(n_1 \ln\left(\dfrac{|\boldsymbol{r}-\boldsymbol{e}_x d|}{l_0}\right) + n_2 \ln\left(\dfrac{r}{l_0}\right) + C_2\right), k_1\right)}, \\ Q = n_1\varphi + n_2\alpha + \varphi_0 \end{cases} \quad (9)$$

where $\boldsymbol{e_x}$ is an ort of the Ox axis and the constants C_1, C_2 should be found from the conditions

$$\begin{cases} c\sqrt{|C_1|}\left(n_1 \ln\left(\dfrac{d+R}{l_0}\right) + n_2 \ln\left(\dfrac{R}{l_0}\right) + C_2\right) = F\left(\arcsin\dfrac{\sqrt{1-b^2}}{k_1}, k_1\right) + 4K(k_1)N_1 \\ c\sqrt{|C_1|}\left(n_1 \ln\dfrac{R}{l_0} + n_2 \ln\left(\dfrac{d-R}{l_0}\right) + C_2\right) = F\left(\arcsin\dfrac{\sqrt{1-b^2}}{k_1}, k_1\right) + 4K(k_1)N_2 \end{cases}, \quad (10)$$

here N_1 and N_2 are integers and $F(\xi, k)$ is the incomplete elliptic integral of the first kind:

$$F(\xi, k) = \int_0^{\xi} \frac{d\rho}{\sqrt{1-k^2\sin^2\rho}}. \quad (11)$$

Systems (9) and (10) determine the sought constants C_1, C_2.

Boundary conditions 2.

On the surface of the antidot 2 $\theta_L = \pi/2$, $\varphi_L = \varphi_0$ for $\alpha = 0$, $\theta_L = \pi/2$ in the point $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$.

In this case, the solution (2), (6) takes an analogous to the previous case form (9). Let us find the distribution constants C_1, C_2. From the boundary conditions on the surface of the antidot 2 we obtain

$$c\sqrt{|C_1|}\left(n_1 \ln\left(\frac{d+R}{l_0}\right) + n_2 \ln\left(\frac{R}{l_0}\right) + C_2\right) = F\left(\arcsin\frac{\sqrt{1-b^2}}{k_1}, k_1\right) + 4K(k_1)N_1, \quad (12)$$

where N_1 is an arbitrary integer. Distribution (6) in the point $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$ has the following form:

$$\begin{cases} P = (n_1 + n_2)\ln\dfrac{d}{2l_0} + C_2 \\ Q = n_2\pi + C_3 \end{cases} \quad (13)$$

hence, using the boundary condition in the point $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$, we obtain

$$c\sqrt{|C_1|}\left((n_1+n_2)\ln\frac{d}{2l_0}+C_2\right)=F\left(\arcsin\frac{\sqrt{1-b^2}}{k_1},k_1\right)+4K(k_1)N_2, \qquad (14)$$

where N_2 is an arbitrary integer. The system (12), (14) determines the constants C_1 and C_2.

Boundary conditions 3.

On the surface of the antidot 2 $\theta_L=\pi/2$ for $\alpha=0$, $\varphi_L=\varphi_0$ and $\theta_L=\pi/2$ in the point $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$

.

From the boundary condition on the surface of the antidot 2 we obtain

$$c\sqrt{|C_1|}\left(n_1\ln\left(\frac{d+R}{l_0}\right)+n_2\ln\left(\frac{R}{l_0}\right)+C_2\right)=F\left(\arcsin\frac{\sqrt{1-b^2}}{k_1},k_1\right)+4K(k_1)N_1, \qquad (15)$$

where N_1 is an arbitrary integer. After taking into account (13), from the boundary condition in the point $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$ implies

$$\begin{cases} c\sqrt{|C_1|}\left((n_1+n_2)\ln\dfrac{d}{2l_0}+C_2\right)=F\left(\arcsin\dfrac{\sqrt{1-b^2}}{k_1},k_1\right)+4K(k_1)N_2 \\ C_3=\varphi_0-n_2\pi \end{cases}, \qquad (16)$$

where N_2 is an arbitrary integer. The system (15), (16) determines the constants C_1, C_2 and C_3.

4. COUPLE OF ANTIDOTS IN AN "EASY PLANE" ANTIFERROMAGNET

Let us select the functions P and Q in the following form:

$$\begin{cases} P=n_1\ln\dfrac{|\boldsymbol{r}|}{l_0}+n_2\ln\left(\dfrac{|\boldsymbol{r}-\boldsymbol{r}_0|}{l_0}\right)+C_2 \\ Q=\dfrac{z}{l_0}+n_1\varphi+n_2\alpha+C_3 \end{cases}, \qquad (17)$$

where α is an azimuthal angle relative to the point $\boldsymbol{r}_0$. Such distribution corresponds, in particular, to a couple of circular antidots in an uniaxial antiferromagnet with the "easy plane" anisotropy. Similarly to the previous case we assume that antidots, in general, are not remote and that their radii R are the same, and we choose the coordinate system in a similar way, so that $\boldsymbol{r}_0=\begin{pmatrix} d \\ 0 \end{pmatrix}$, $d>0$. Thus, the functions P and Q on the antidots surfaces have the following form:

$$\begin{cases} P = n_1 \ln \dfrac{R}{l_0} + n_2 \ln\left(\dfrac{d}{l_0} \sqrt{1 + \dfrac{R^2}{d^2} - 2\dfrac{R}{d}\cos\varphi} \right) + C_2 \\ Q = \dfrac{z}{l_0} + n_1\varphi + n_2\left(\pi - arctg \dfrac{\sin\varphi}{d/R - \cos\varphi} \right) + C_3 \end{cases} \quad (18)$$

on the surface of the antidot 1, and

$$\begin{cases} P = n_1 \ln\left(\dfrac{d}{l_0} \sqrt{1 + \dfrac{R^2}{d^2} + 2\dfrac{R}{d}\cos\alpha} \right) + n_2 \ln\left(\dfrac{R}{l_0} \right) + C_2 \\ Q = \dfrac{z}{l_0} + n_1 arctg\left(\dfrac{\sin\alpha}{d/R + \cos\alpha} \right) + n_2\alpha + C_3 \end{cases} \quad (19)$$

on the surface of the antidot 2.

As we can see, because of the dependence of the function Q on z we cannot set constant boundary conditions for φ_L on some interval $\begin{pmatrix} r_1 \\ \varphi_1 \end{pmatrix}$. However, we can set boundary conditions in some point $\begin{pmatrix} r_1 \\ \varphi_1 \\ z_1 \end{pmatrix}$, and for the sake of simplicity we can choose the origin of the Oz axis in the plane $z = z_1$. Let us illustrate this approach on three different sets of boundary conditions, analogous to the previous (antidots couple in an isotropic antiferromagnet) case.

Boundary conditions 1.

On the surface of the antidot 2 $\theta_L = \pi/2$ for $\alpha = 0$, $\varphi_L = \varphi_0$ for $\alpha = 0$, $z = 0$; on the surface of the antidot 1 $\theta_L = \pi/2$ for $\varphi = 0$.

From the boundary condition for φ_L we obtain $C_3 = \varphi_0$, so the solution (2), (17) in this case can be rewritten

$$\begin{cases} tg\left(\dfrac{\theta_L}{2} \right) = \dfrac{b}{dn\left(c\sqrt{|C_1|}\left(n_1 \ln\left(\dfrac{|\boldsymbol{r} - \boldsymbol{e}_x d|}{l_0} \right) + n_2 \ln\left(\dfrac{r}{l_0} \right) + C_2 \right), k_1 \right)} \\ Q = \dfrac{z}{l_0} + n_1\varphi + n_2\alpha + \varphi_0 \end{cases} \quad (20)$$

with constants C_1, C_2 determined from the conditions similar to the previous case:

$$\begin{cases} c\sqrt{|C_1|}\left(n_1\ln\left(\dfrac{d+R}{l_0}\right)+n_2\ln\left(\dfrac{R}{l_0}\right)+C_2\right)=F\left(\arcsin\dfrac{\sqrt{1-b^2}}{k_1},k_1\right)+4K(k_1)N_1 \\ c\sqrt{|C_1|}\left(n_1\ln\dfrac{R}{l_0}+n_2\ln\left(\dfrac{d-R}{l_0}\right)+C_2\right)=F\left(\arcsin\dfrac{\sqrt{1-b^2}}{k_1},k_1\right)+4K(k_1)N_2 \end{cases}, \quad (21)$$

here N_1, N_2 are arbitrary integers. Systems (20) and (21) determine the sought solution for the boundary conditions we consider.

Boundary conditions 2.

On the surface of the antidot 2 $\theta_L = \pi/2$ for $\alpha = 0$, $\varphi_L = \varphi_0$ for $\alpha = 0$, $z = 0$; $\theta_L = \pi/2$ in the point $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$.

In an analogous way to the previous boundary condition choice we obtain $C_3 = \varphi_0$, so the solution (2), (17) takes the form (20). Distribution constants C_1, C_2 are found analogously to the previous case, boundary conditions 2: from the boundary conditions on the surface of antidot 2 we obtain

$$c\sqrt{|C_1|}\left(n_1\ln\left(\frac{d+R}{l_0}\right)+n_2\ln\left(\frac{R}{l_0}\right)+C_2\right)=F\left(\arcsin\frac{\sqrt{1-b^2}}{k_1},k_1\right)+4K(k_1)N_1, \quad (22)$$

where N_1 is an arbitrary integer, and from the boundary conditions in the point $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$ we obtain

$$c\sqrt{|C_1|}\left((n_1+n_2)\ln\frac{d}{2l_0}+C_2\right)=F\left(\arcsin\frac{\sqrt{1-b^2}}{k_1},k_1\right)+4K(k_1)N_2, \quad (23)$$

where N_2 is an arbitrary integer. The system (20), (22), (23) determines the sought solution for the boundary conditions that we consider.

Boundary conditions 3.

On the surface of the antidot 2 $\theta_L = \pi/2$ for $\alpha = 0$, $\theta_L = \pi/2$ on the interval $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$, $\varphi_L = \varphi_0$ on the same interval in the point $z = 0$.

In an analogous way, from the boundary conditions on the surface of the antidot 2 we obtain

$$c\sqrt{|C_1|}\left(n_1\ln\left(\frac{d+R}{l_0}\right)+n_2\ln\left(\frac{R}{l_0}\right)+C_2\right)=F\left(\arcsin\frac{\sqrt{1-b^2}}{k_1},k_1\right)+4K(k_1)N_1, \quad (24)$$

where N_1 is an arbitrary integer. Similarly to the previous case, for the function P the relation $P = (n_1 + n_2)\ln\frac{d}{2l_0} + C_2$ fulfils; from this relation,

$$c\sqrt{|C_1|}\left((n_1 + n_2)\ln\frac{d}{2l_0} + C_2\right) = F\left(\arcsin\frac{\sqrt{1-b^2}}{k_1}, k_1\right) + 4K(k_1)N_2 \qquad (25)$$

implies, where N_2 is an arbitrary integer. From the boundary condition for φ_L we obtain

$$C_3 = \varphi_0 - n_2\pi\,. \qquad (26)$$

The system (24), (25), (26) determines the constants C_1, C_2, C_3, and together with (20) determines the sought solution.

5. COUPLE OF ANTIDOTS IN AN "EASY AXIS" ANTIFERROMAGNET

Let us select the functions P and Q in the following form:

$$\begin{cases} P = \dfrac{z}{l_0} + n_1\ln\dfrac{|\boldsymbol{r}|}{l_0} + n_2\ln\left(\dfrac{|\boldsymbol{r}-\boldsymbol{r}_0|}{l_0}\right) + C_2 \\ Q = n_1\varphi + n_2\alpha + C_3 \end{cases} \qquad (27)$$

where α is an azimuthal angle relative to the point $\boldsymbol{r}_0$. Such distribution corresponds, in particular, to a couple of circular antidots in an uniaxial antiferromagnet with the antiferromagnet of the "easy axis" type anisotropy. Similarly to the previous case we assume that antidots, in general, are not remote and that their radii R are the same, and we choose the coordinate system in an analogous way, so that $\boldsymbol{r}_0 = \begin{pmatrix} d \\ 0 \end{pmatrix}$, $d > 0$. Therefore, the functions P and Q on the antidots surfaces can be written as follows:

$$\begin{cases} P = \dfrac{z}{l_0} + n_1\ln\dfrac{R}{l_0} + n_2\ln\left(\dfrac{d}{l_0}\sqrt{1 + \dfrac{R^2}{d^2} - 2\dfrac{R}{d}\cos\varphi}\right) + C_2 \\ Q = n_1\varphi + n_2\left(\pi - arctg\dfrac{\sin\varphi}{d/R - \cos\varphi}\right) + C_3 \end{cases} \qquad (28)$$

on the surface of the antidot 1, and

$$\begin{cases} P = \dfrac{z}{l_0} + n_1\ln\left(\dfrac{d}{l_0}\sqrt{1 + \dfrac{R^2}{d^2} + 2\dfrac{R}{d}\cos\alpha}\right) + n_2\ln\left(\dfrac{R}{l_0}\right) + C_2 \\ Q = n_1 arctg\left(\dfrac{\sin\alpha}{d/R + \cos\alpha}\right) + n_2\alpha + C_3 \end{cases} \qquad (29)$$

on the surface of the antidot 2. Analogously to the previous case we cannot set the boundary conditions on the interval $\begin{pmatrix} r_1 \\ \varphi_1 \end{pmatrix}$, but this time because of the dependence of the function θ_L on z, so we have to set at boundary conditions for the function θ_L in some point. In a similar way to the previous case, it is convenient to choose the origin of the axis Oz in this point. Note that because of the presence of two constants C_1 and C_2 in the general form of the function θ_L, in order to determine them we have to specify the function θ_L in two points, and for the sake of convenience we can choose two points with the same z-coordinate and set the origin of the Oz axis in these points. Having made these remarks, we can now find the distribution of the antiferromagnetic vector in an analogous to the previous two cases way. Let us consider three analogous boundary conditions sets.

Boundary conditions 1.

On the surface of the antidot 2 $\theta_L = \pi/2$ for $\alpha = 0$, $z = 0$, $\varphi_L = \varphi_0$ for $\alpha = 0$; on the surface of the antidot 1 $\theta_L = \pi/2$ for $\varphi = 0$, $z = 0$.

Solution (2), (27) in this case can be rewritten as

$$\begin{cases} tg\left(\dfrac{\theta_L}{2}\right) = \dfrac{b}{dn\left(c\sqrt{|C_1|}\left(\dfrac{z}{l_0} + n_1 \ln\left(\dfrac{|\boldsymbol{r} - \boldsymbol{e}_x d|}{l_0}\right) + n_2 \ln\left(\dfrac{r}{l_0}\right) + C_2\right), k_1\right)} \\ Q = n_1\varphi + n_2\alpha + \varphi_0 \end{cases} . \quad (30)$$

After substituting the boundary conditions into it we can see that the distribution constants C_1, C_2 are determined from the same system (21) as in the previous case.

Boundary conditions 2.

On the surface of the antidot 2 $\theta_L = \pi/2$ for $\alpha = 0$, $z = 0$, $\varphi_L = \varphi_0$ for $\alpha = 0$; $\theta_L = \pi/2$ in the point $\begin{pmatrix} d/2 \\ 0 \\ 0 \end{pmatrix}$.

From the boundary conditions for φ_L in a similar way to the previous boundary conditions choice we obtain $C_3 = \varphi_0$, so that the solution of (2), (27) takes the form (30). The system for determining the distribution constants C_1, C_2 has the same form as for the boundary conditions 2 in the previous case, so these constants are determined from the system (22), (23).

Boundary conditions 3.

On the surface of the antidot 2 $\theta_L = \pi/2$ for $\alpha = 0$, $z = 0$; $\varphi_L = \varphi_0$ on the interval $\begin{pmatrix} d/2 \\ 0 \end{pmatrix}$ and $\theta_L = \pi/2$ on the same interval in the point $z = 0$.

After substituting these boundary conditions into the solution (2), (27), we can see that despite of the different type of the antiferromagnet and the different form (27) of the distribution functions, constants of this distribution are determined by the similar to the previous case system (24), (25), (26).

6. PERIODIC ANTIDOT SYSTEM IN AN ISOTROPIC ANTIFERROMAGNET

Let us select the functions P and Q in the following form:

$$\begin{cases} P = \sum_i n_i \ln\left(\dfrac{|\boldsymbol{r} - \boldsymbol{r}_{0i}|}{l_0}\right) + C_2 \\ Q = \sum_i \alpha_i n_i + C_3 \end{cases}. \tag{31}$$

Such distribution corresponds, in particular, to a system of antidots with the centers $\{\boldsymbol{r}_{0i}\}$ in a film composed of an isotropic antiferromagnet.

We assume that the radii of the antidots are all equal and equal to R. Let us consider the case when antidots in the antiferromagnet form a periodic structure with the period a along the Ox axis and the period b along the Oy axis. Therefore, the relation

$$\boldsymbol{r}_{0i} = \boldsymbol{r}_0 + a\boldsymbol{e}_x p_i + b\boldsymbol{e}_y q_i \tag{32}$$

is satisfied, here p_i, q_i are integers. Because of the translational symmetry of the system all the factors n_i in the distribution (31) are equal: $n_i = n$ for any i.

Note that because of the periodicity of the elliptic functions we cannot consider the infinite sum of logarithms in the expression for the function P. However, we can consider the number of the antidots limited, but still large enough so that we can use the translational symmetry condition.

Let us use constant boundary conditions on the surface of some antidot: θ_L and φ_L are constant when $\left|\boldsymbol{r} - \boldsymbol{r}_{0i_0}\right| = R$ for the antidot i_0. Note that in order to use such boundary condition we should consider a periodic system of antidots that are distant from each other. Indeed, after substituting such boundary condition into the solution (31) we obtain the equation that contains a variable addend $\sum_{i \neq i_0} n_i \ln\left(\dfrac{|\boldsymbol{r} - \boldsymbol{r}_{0i}|}{l_0}\right)$ that depends on the point of the antidot surface. However, if the antidots are distant from one another, so that the condition $\left|\boldsymbol{r}_{0i} - \boldsymbol{r}_{0i_0}\right| >> R$ is fulfilled, this addend can be considered approximately constant and we can write down the following relation on the surface of the antidot i_0:

$$P \approx n \ln\frac{R}{l_0} + n\sum_{i \neq i_0} \ln\left(\frac{\left|\boldsymbol{r}_{0i} - \boldsymbol{r}_{0i_0}\right|}{l_0}\right) + C_2 = n \ln\frac{R}{l_0} + \widetilde{C}_2, \tag{33}$$

here $\widetilde{C}_2 = n\sum_{i \neq i_0} \ln\left(\dfrac{\left|\boldsymbol{r}_{0i} - \boldsymbol{r}_{0i_0}\right|}{l_0}\right) + C_2 \approx const$. Note that these considerations are correct for any antidot system with the antiferromagnetic vector distribution determined by (31), not necessarily a periodic system.

In a similar way, variable addend $\sum_{i\neq i_0}\alpha_i\left(\boldsymbol{r}_{i_0}\right)n_i$ enters boundary conditions for φ_L. This addend can be neglected provided $\sum_{i\neq i_0}\Delta\alpha\left(i,i_0\right)<<\frac{\pi}{2}$, where $\Delta\alpha\left(i,i_0\right)$ is the maximum difference between the angles α_i of the antidot i_0 axis and an arbitrary point on the antidot i_0 surface. In this case, due to the symmetry of the problem the sum $\sum_{i\neq i_0}\alpha_i\left(\boldsymbol{r}_{i_0}\right)n_i$ is a multiple of 2π for any point on the surface of the antidot i_0, therefore, for such antidots remoteness condition the following relation fulfils:

$$Q\left(\left|\boldsymbol{r}-\boldsymbol{r}_{0i_0}\right|=R\right)\approx n\alpha_{i_0}+C_3\,. \tag{34}$$

Thus, we can impose arbitrary constant boundary conditions on the surface of each antidot as long as these conditions are the same for all antidots. For example, we can choose the boundary conditions as follows:

$$\begin{cases}\theta_L\left(\left|\boldsymbol{r}-\boldsymbol{r}_{0i_0}\right|=R\right)=\dfrac{\pi}{2}\\ \varphi_L\left(\left|\boldsymbol{r}-\boldsymbol{r}_{0i_0}\right|=R\right)=\alpha_{i_0}+\dfrac{\pi}{2}\pm\pi\end{cases}. \tag{35}$$

These boundary conditions correspond to a positive vortex distribution of the antiferromagnetic vector in the xy plane on the surface of each antidot. In this case $C_3=\frac{\pi}{2}\pm\pi$, and the constant C_1 can be determined from the condition

$$c\sqrt{\left|C_1\right|}\ln\left(\frac{R}{l_0}\right)=F\left(\arcsin\frac{\sqrt{1-b^2}}{k_1},k_1\right)+4K\left(k_1\right)N\,. \tag{36}$$

Here we put $\widetilde{C}_2=0$ (for a complete determination of the distribution we need either to impose another boundary condition in addition to (35) or set one of the constants $C_1,\widetilde{C}_2$).

Note that the expression for θ_L can be simplified considering the fact that if the case of the antidots remoteness is fulfilled, it can approximately be reduced to the sum over the four nearest antidots:

$$\begin{aligned}P&=n\sum_i\ln\left(\frac{\left|\boldsymbol{r}-\boldsymbol{r}_{0i}\right|}{l_0}\right)+C_2\approx n\sum_{|x-x_i|\le a,|y-y_i|\le b}\ln\left(\frac{\left|\boldsymbol{r}-\boldsymbol{r}_{0i}\right|}{l_0}\right)+\widetilde{C}_2-\\&-n\left(\ln\left(\frac{a}{l_0}\right)+\ln\left(\frac{b}{l_0}\right)+\ln\left(\frac{\sqrt{a^2+b^2}}{l_0}\right)\right)=\\&=n\left(\sum_{|x-x_i|\le a,|y-y_i|\le b}\ln\left(\frac{\left|\boldsymbol{r}-\boldsymbol{r}_{0i}\right|}{l_0}\right)-\ln\left(\frac{a}{l_0}\right)-\ln\left(\frac{b}{l_0}\right)-\ln\left(\frac{\sqrt{a^2+b^2}}{l_0}\right)\right)+\widetilde{C}_2\end{aligned} \tag{37}$$

In particular, for $\widetilde{C}_2 = 0$ the required distribution can be written

$$\begin{cases} b \cdot ctg\left(\frac{\theta_L}{2}\right) = dn\left(cn\sqrt{|C_1|}\left(\sum_{|x-x_i|\le a, |y-y_i|\le b} \ln\left(\frac{|\boldsymbol{r}-\boldsymbol{r}_{0i}|}{l_0}\right) - \ln\left(\frac{a}{l_0}\right) - \ln\left(\frac{b}{l_0}\right) - \right.\right. \\ \left.\left. - \ln\left(\frac{\sqrt{a^2+b^2}}{l_0}\right)\right), k_1 \right) \\ \varphi_L = n\sum_i \alpha_i + \frac{\pi}{2} \pm \pi \end{cases} \quad (38)$$

with the constant C_1 determined from the condition (36).

Note that the distribution we obtained is correct only far from the boundaries of the antidot system, so the approximation of the translational symmetry can be applied.

CONCLUSIONS

Thus, we wrote down a solution of the Landau-Lifshitz equation for an antidot system in a film composed of an uniaxial or anisotropic two-sublattice antiferromagnet. Using this solution, we have found an antiferromagnetic vector configuration for a periodic remote antidot system in an isotropic antiferromagnet and for an antidot couple (in general, not a remote couple) in an isotropic antiferromagnet, in an antiferromagnet with uniaxial anisotropy of the "easy plane" type and in an antiferromagnet with uniaxial anisotropy of the "easy axis" type for three different variants of boundary conditions. We have shown that for an antidot system in a film composed of an uniaxial or isotropic two-sublattice antiferromagnet, constant boundary conditions on a surface of some (arbitrary) antidot is, in general, impossible, however, such boundary conditions are possible, for example, for a remote antidot system in an isotropic antiferromagnet.

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Горобець Ю. І. Розподіл вектора антиферомагнетизму для періодичної системи віддалених кругових антиточок у ізотропному антиферомагнетику та пари кругових антиточок у ізотропному або одновісному антиферомагнетику / Ю. І. Горобець, О. Ю. Горобець, В. В. Куліш // Вчені записки Таврійського національного університету імені В. И. Вернадського. Серія : Фізико-математичні науки. – 2013. – Т. 26 (65), № 2. – С. 38-52.

Робота присвячена теоретичному дослідженню розподілу вектора антиферомагнетизму у плівці з ізотропного та одновісного антиферомагнетиків при наявності системи кругових антиточок. Отримано розв'язок рівняння Ландау-Ліфшица для вектора антиферомагнетизму у такій системі антиточок. Для періодичної системи віддалених антиточок у ізотропному антиферомагнетику, а також для пари антиточок (взагалі, не віддалених) у ізотропному, легковісному та легкоплощинному антиферомагнетику отримано розподіл вектора антиферомагнетизму за різних граничних умов.

Ключові слова: антиферомагнетик, магнітна тонка плівка, магнітна антиточка, вектор антиферомагнетизму.

Горобец Ю. И. Распределение вектора антиферромагнетизма для периодической системы удаленных круговых антиточек в изотропном антиферромагнетике и пары круговых антиточек в изотропной или одноосном антиферромагнетике / Ю. И. Горобец, О. Ю. Горобец, В. В. Кулиш // Ученые записки Таврического национального университета имени В. И. Вернадского. Серия : Физико-математические науки. – 2013. – Т. 26 (65), № 2. – С. 38-52.

Работа посвящена теоретическому исследованию распределения вектора антиферромагнетизма в пленке из изотропного и одноосного антиферромагнетиков при наличии системы круговых антиточек. Получено решение уравнения Ландау-Лифшица для вектора антиферромагнетизма в такой системе антиточек. Для периодической системы удаленных антиточек в изотропном антиферромагнетике, а также для пары антиточек (вообще, не удаленных) в изотропном, легкоосном и легкоплоскостном антиферромагнетике получено распределение вектора антиферромагнетизма при различных граничных условиях.

Ключевые слова: антиферромагнетик, магнитная тонкая пленка, магнитная антиточка, вектор антиферромагнетизма.

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