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SCALAR FIELD POTENTIAL DISTRIBUTION FOR TWO A “THICK” NULL STRING MOVING ALONG THE AXIS-Z

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The general form of the scalar field potential distribution for two a “thick” null string moving along the axis-Z and completely lying in a plane orthogonal to this axis at every time moment is proposed. The conditions, under which a contraction of the field to a one-dimensional object results asymptotic coincidence of components of the energy-momentum tensor in the limit of compression components energy-momentum tensor of a scalar field for a system of two noninteracting null strings moving on the same trajectories are found.

Keywords: null string, scalar field, cosmology.

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INTRODUCTION

String theories show the steady progress during several recent decades. In spite of problems inevitable for any developing theory, they arouse admiration both due to the results already obtained and their great possibilities in the future. On the one hand, the interest in cosmic strings and other topological solutions is initiated by the role possibly played by topological defects in the process of evolution of the Universe (string mechanisms of generation of primary inhomogeneities of the matter density in the early Universe or ideas of the topological inflation). On the other hand, it is due to the physical properties of these objects significantly differing from those of common matter.

Besides studying of string theory allows us to understand the deepest moments of the birth of the Universe in order to understand why it occurred, and what lies ahead of her? But it is impossible to imagine studying the evolution of the Universe without studying the properties of its components. That's why this article is a studying of null strings, which are an integral part of both the string theory and the universe in general.

Objective of article:

- Construct the general view of distribution of potential scalar field for a system consisting of two "thick" null string moving without interaction along the axis in the same direction.
- Find conditions on scalar field potential at which, within the limit of the compression component of the energy-momentum tensor of the scalar field asymptotically coincide with the components of the energy-momentum tensor for a system of two null strings moving on the same trajectory.

The components of the energy-momentum tensor for an isolated null strings have the following form:

$$T^{mn}\sqrt{-g}=\gamma\int d\tau d\sigma x^m_{,\tau}x^n_{,\tau}\delta^4\left(x^l-x^l\left(\tau,\sigma\right)\right), \tag{1}$$

where the indices m,n,l take the values 0,1,2,3, functions $x^m=x^m\left(\tau,\sigma\right)$ determine the trajectory of a null string, τ and σ are the parameters on the light surface of the null strings $x^m_{,\tau}=\partial x^m/\partial\tau$, $g=\left|g_{mn}\right|$, g_{mn} is the metric tensor of the environment, and $\gamma=const$.

In the cylindrical system of coordinates: $x^0=t$, $x^1=\rho$, $x^2=\theta$, $x^3=z$, the function $x^m\left(\tau,\sigma\right)$, that determine the trajectory of the two closed string with constant (time-invariant) radius R, moving along in the negative direction the axis z completely lying in a plane orthogonal to this axis at every time moment have the following form:

$$t=\tau,\ \rho=R=const.,\ \theta=\sigma,\ z=a-\tau, \tag{2}$$

$$t=\tau,\ \rho=R=const.,\ \theta=\sigma,\ z=-a-\tau, \tag{3}$$

where the positive constant a determines the distance, to the variable z, between the two null strings (equal $2a$).

For trajectories (2), (3), all directions on the hypersurfaces $z=const$ are equivalent; therefore, the metric functions $g_{mn}=g_{mn}(t,\rho,z)$, using the invariance of the quadratic form with respect to the inversion of θ to $-\theta$ we obtain $g_{02}=g_{12}=g_{32}=0$. One can also see that the space-time quadratic form must be invariant with respect to the simultaneous inversion $t\to -t,\ z\to -z$. Hence,

$$g_{mn}(t,\rho,z)=g_{mn}(-t,\rho,-z), \tag{4}$$

which yields

$$g_{01}=g_{31}=0. \tag{5}$$

Finally, using the free choice of the systems of coordinates in the general relativity theory, we partially fix it by the requirement

$$g_{30}=0. \tag{6}$$

Thus, the quadratic form for the problem to be solved can be presented as

$$dS^2=e^{2\nu}(dt)^2-A(d\rho)^2-B(d\theta)^2-e^{2\mu}(dz)^2, \tag{7}$$

where ν,μ,A,B depend on the variables t,ρ,z.

The components of the energy-momentum tensor for a system of two non-interacting null strings moving along the trajectories of (2) and (3) have the following form:

$$T^{mn} = \underset{1}{T}^{mn} + \underset{2}{T}^{mn}, \tag{8}$$

where $\underset{1}{T}^{mn}$ and $\underset{2}{T}^{mn}$ components of energy-momentum tensor for an isolated null string moving along a path (2) and (3) respectively. For massless components of the energy-momentum must satisfy the equation

$$T_\alpha^\alpha = 0. \tag{9}$$

Hence, to (1), (2), (3), (7), (8), Eq. (9) takes the form:

$$T_0^0 + T_3^3 = \frac{2\gamma}{\sqrt{AB}}\left\{e^{\nu-\mu} - e^{\mu-\nu}\right\}\left\{\delta(q+a) + \delta(q-a\right\}\delta(\rho - R) = 0, \tag{10}$$

whence

$$\nu \equiv \mu. \tag{11}$$

The non-zero components of the energy-momentum tensor (1), to (1) – (3), (7), (11), are as follows:

$$T_{00} = T_{33} = T_{03} = \gamma\frac{e^{-2\nu}}{\sqrt{AB}}\delta(\rho - R)\big(\delta(q+a) + \delta(q-a)\big) \tag{12}$$

Analyzing the system of Einstein equations and using conditions for (7), (11), (12) the dependence of functions of the quadratic form (7) can be redefined as

$$A = A(q,\rho), \quad B = B(q,\rho), \quad \nu = \nu(q,\rho), \tag{13}$$

where $q = t + z$.

In this case, the Einstein system itself is reduced to the equations

$$-\frac{A_{,qq}}{2A} - \frac{B_{,qq}}{2B} + \frac{1}{4}\left(\left(\frac{A_{,q}}{A}\right)^2 + \left(\frac{B_{,q}}{B}\right)^2\right) + \nu_{,q}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) = \chi T_{00}, \tag{14}$$

$$2\nu_{,\rho\rho} + 2\left(\nu_{,\rho}\right)^2 + \frac{B_{,\rho\rho}}{B} - \frac{1}{2}\left(\frac{B_{,\rho}}{B}\right)^2 + \nu_{,\rho}\left(\frac{B_{,\rho}}{B} - \frac{A_{,\rho}}{A}\right) - \frac{1}{2}\frac{A_{,\rho}}{A}\frac{B_{,\rho}}{B} = 0, \tag{15}$$

$$\left(\nu_{,\rho}\right)^2 + \nu_{,\rho}\frac{B_{,\rho}}{B} = 0, \tag{16}$$

$$2\nu_{,\rho\rho} + 3\left(\nu_{,\rho}\right)^2 - \nu_{,\rho}\frac{A_{,\rho}}{A} = 0, \qquad (17)$$

$$\frac{B_{,q\rho}}{B} + 2\nu_{,q\rho} - \nu_{,\rho}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) - \frac{1}{2}\frac{B_{,\rho}}{B}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) = 0. \qquad (18)$$

Supplement the system (14) - (18) the motion of a null string in the pseudo-Riemannian space

$$x^m_{,\tau\tau} + \Gamma^m_{pq} x^p_{,\tau} x^q_{,\tau} = 0, \qquad (19)$$

$$g_{mn} x^m_{,\tau} x^n_{,\tau} = 0, \; g_{mn} x^m_{,\tau} x^n_{,\sigma} = 0, \qquad (20)$$

where Γ^m_{pq} are the Christoffel symbols. It can be shown on that the trajectory (2), (3), equation of motion (19), (20) are performed identically, i.e. the trajectory of (2), (3) are actually realized and do not change the gravitational field itself null strings.

Eq. (12) implies that, beyond the two strings, i.e. at $q \neq \pm a, \;\; \rho \neq R$, all components of its energy-momentum tensor are equal to zero, while the non-zero ones (tending to infinity) appear directly at the string, this allows one to investigate the system of Einstein equations (14) - (18) in two directions:

1. By restricting oneself to the analysis of "external" problem in the region $q \neq \pm a, \;\; \rho \neq R,$ where the components of the energy-momentum tensor (right-hand sides of the Einstein equations (14) - (18)) are equal to zero.
2. By considering the components of the energy-momentum tensor of a string as a limit of some "thick" distribution and analyzing the Einstein equations for this "thick" distribution.

Can be shown, the analysis of the "external" problem results in a large number of vacuum solutions of Einstein equations (14) - (18) that satisfy the problem symmetry, however, the criteria allowing one to choose those describing the gravitational field of a null string from this totality of solutions remain unclear. For example, it is easy to check that the function

$$e^{2\nu} = A = 1, \;\; B = \rho^2, \qquad (21)$$

defining the Minkowski space-time, or function

$$e^{2\nu} = c\left|\beta_{,q}\right|, \;\; A = \left(\beta(q)\right)^2, \;\; B = \left(\beta(q)\rho\right)^2, \qquad (22)$$

$c = const.$, $\beta(q)$ arbitrary function, there are external solutions (i.e. in the region $q \neq \pm a, \;\; \rho \neq R$) of Einstein equations (14) - (18).

When trying to consider the components of the tensor (12) of the string as a limit of some “thick” distribution (simply replacing the delta functions in the tensor (12) by the corresponding delta-function sequences), some errors can arise due to the indeterminacy of considering the possible appearance of terms (multipliers) tending to zero (constant) under the contraction of this"thick" distribution into a one-dimensional object. That is why it is more suitable to start from some "well- determined" "thick" distribution, such as a real massless scalar field (as we consider a scalar null object) and then to contract it to a string of the required configuration provided that the components of the tensor (12) of the scalar field asymptotically coincide with those of the components of the tensor (12).

1. SYSTEM OF EINSTEIN EQUATIONS FOR THE "THICK" PROBLEM

The components of the energy-momentum tensor for a real massles scalar field have the form:

$$T_{\alpha\beta}=\varphi_{,\alpha}\varphi_{,\beta}-\frac{1}{2}g_{\alpha\beta}L, \tag{23}$$

where $L=g^{\omega\lambda}\varphi_{,\omega}\varphi_{,\lambda}$, $\varphi_{,\alpha}=\partial\varphi/\partial x^{\alpha}$, φ is the scalar field potential, and this indices $\alpha,\beta,\omega,\lambda$ take on the values $0,1,2,3$.

To provide the self-consistency of the Einstein equations constructed for (14) - (18) for the tensor (23), we demand that:

$$T_{\alpha\beta}=T_{\alpha\beta}(q,\rho)\rightarrow\varphi=\varphi(q,\rho). \tag{24}$$

System Einstein equation for (7), (11) (13) (24) (24) can be represented as follows

$$-\frac{1}{2}\left(\frac{A_{,qq}}{A}+\frac{B_{,qq}}{B}\right)+\nu_{,q}\left(\frac{A_{,q}}{A}+\frac{B_{,q}}{B}\right)+\frac{1}{4}\left(\left(\frac{A_{,q}}{A}\right)^2+\left(\frac{B_{,q}}{B}\right)^2\right)=\chi\left(\varphi_{,q}\right)^2, \tag{25}$$

$$2\nu_{,\rho\rho}+2\left(\nu_{,\rho}\right)^2+\frac{B_{,\rho\rho}}{B}-\frac{1}{2}\left(\frac{B_{,\rho}}{B}\right)^2+ +\nu_{,\rho}\left(\frac{B_{,\rho}}{B}-\frac{A_{,\rho}}{A}\right)-\frac{1}{2}\frac{A_{,\rho}}{A}\frac{B_{,\rho}}{B}=-\chi\left(\varphi_{,\rho}\right)^2, \tag{26}$$

$$\left(\nu_{,\rho}\right)^2+\nu_{,\rho}\frac{B_{,\rho}}{B}=\frac{\chi}{2}\left(\varphi_{,\rho}\right)^2, \tag{27}$$

$$2\nu_{,\rho\rho}+3\left(\nu_{,\rho}\right)^{2}-\nu_{,\rho}\frac{A_{,\rho}}{A}=-\frac{\chi}{2}\left(\varphi_{,\rho}\right)^{2}, \tag{28}$$

$$-\frac{1}{2}\frac{B_{,q\rho}}{B}-\nu_{,q\rho}+\frac{1}{2}\nu_{,\rho}\left(\frac{A_{,q}}{A}+\frac{B_{,q}}{B}\right)+\frac{1}{4}\frac{B_{,\rho}}{B}\left(\frac{A_{,q}}{A}+\frac{B_{,q}}{B}\right)=\chi\varphi_{,q}\varphi_{,\rho}. \tag{29}$$

Let us consider system (25) - (29) for the distribution of the scalar field already concentrated inside a “thin” ring, with the variables q and ρ taking values in the interval:

$$q\in\left(-a-\Delta q,-a+\Delta q\right)\cup(a-\Delta q,a+\Delta q),\ \ \rho\in\left(R-\Delta\rho,\ R+\Delta\rho\right), \tag{30}$$

where Δq and $\Delta\rho$ are small positive constants, determine the "thickness" of the rings,

$$\Delta q<<1,\ \Delta\rho<<1, \tag{31}$$

with a further contraction of this "thin" rings into one-dimensional objects (null string)

$$\Delta q\to 0,\ \Delta\rho\to 0, \tag{32}$$

the space, where two such "thick" null strings moves and for which the variables q and ρ ake on values in the interval

$$q\in\left(-\infty,+\infty\right),\ \rho\in\left[0,+\infty\right), \tag{33}$$

can be conditionally divided into three regions:
- region I, for which

$$q\in\left(-\infty,-a-\Delta q\right)\cup(-a+\Delta q,a-\Delta q)\cup\left(a+\Delta q,+\infty\right),\rho\in\left[0,+\infty\right), \tag{34}$$

- region II, for which

$$q\in(-a-\Delta q,-a+\Delta q)\cup(a-\Delta q,a+\Delta q),\ \rho\in\left[0,\ R-\Delta\rho\right)\cup\left(R+\Delta\rho,\ +\infty\right) \tag{35}$$

- region III, for which

$$q\in[-a-\Delta q,-a+\Delta q)\cup(a-\Delta q,a+\Delta q],\ \ \rho\in\left[R-\Delta\rho,\ R+\Delta\rho\right], \tag{36}$$

Since the contraction of the scalar field into a string mast result in the asymptotic coincidence of system (24) – (28) with the system for a closed null string (14) – (18) we obtain for the regions I, II

$$\varphi\to 0,\ \ \varphi_{,\rho}\to 0,\ \ \varphi_{,q}\to 0, \tag{37}$$

for the region III, in the general case,

$$\frac{\varphi_{I,II}}{\varphi_{III}} \le 1, \quad \frac{\left(\varphi_{,\rho}\right)_{I,II}}{\left(\varphi_{,\rho}\right)_{III}} \le 1, \quad \frac{\left(\varphi_{,q}\right)_{I,II}}{\left(\varphi_{,q}\right)_{III}} \le 1, \tag{38}$$

where $\varphi_{I,II}$ are values of the scalar field potential in the regions I, II, φ_{III} are values of the scalar field potential in the region III (inside the "thin" ring), equality is realized on the boundary, i.e. at

$$q \in [-a-\Delta q, -a+\Delta q) \cup (a-\Delta q, a+\Delta q], \ \ \rho \to R \pm \Delta\rho, \tag{39}$$

$$\rho \in \left[R-\Delta\rho, \ R+\Delta\rho\right], \ \ q \to \mp a \pm \Delta q. \tag{40}$$

Comparing the system of Einstein equations for a closed null strings (14) - (18) t with system (24) - (28) we may conclude that, under the contraction of the scalar field into a string of the required configuration, i.e., at $\Delta q \to 0, \ \Delta\rho \to 0$

$$\left.\left(\varphi_{,\rho}\right)^2\right|_{q\to\mp a, \rho\to R} \to 0, \ \left.\left(\varphi_{,q}\right)^2\right|_{q\to\mp a, \rho\to R} \to \infty, \ \left.\left(\varphi_{,q}\varphi_{,\rho}\right)\right|_{q\to\mp a, \rho\to R} \to 0, \tag{41}$$

In the region I, for any fixed value of the variable $q = q_0 \in \left(-\infty, -a-\Delta q\right) \cup (-a+\Delta q, a-\Delta q) \cup \left(a+\Delta q, \ +\infty\right)$ and for all values of the variable $\rho \in \left[0, +\infty\right)$, the potential of the scalar field:

$$\varphi(\mathrm{q}_0, \rho) \to 0, \tag{42}$$

according to (37), the scalar field potential in region I at any fixed value of $q = \mathrm{q}_0 \in (-a-\Delta q, -a+\Delta q) \cup (a-\Delta q, a+\Delta q)$,(region II and III), in the case where the variable $\rho \in \left[0, \ R-\Delta\rho\right) \cup \left(R+\Delta\rho, \ +\infty\right)$ (region II), must be performed:

$$\varphi(\mathrm{q}_0, \rho) \to 0, \tag{43}$$

whereas, for $\rho \in \left(R-\Delta\rho, \ R+\Delta\rho\right)$ (region III):

$$\frac{\varphi(\mathrm{q}_0, \rho)_{III}}{\varphi(\mathrm{q}_0, \rho)_{II}} > 1. \tag{44}$$

2. SCALAR FIELD POTENTIAL DISTRIBUTION FOR A «THICK» NULL STRING

For the conditions (42) - (44) it is suitable to present the scalar field potential distribution in the form:

$$\varphi(q,\rho)=\ln\left(\frac{1}{\alpha(q)+\lambda(q)f(\rho)}\right), \tag{45}$$

where the function $\alpha(q)$ and $\lambda(q)$ are symmetric with respect to the inversion of q on $-q$:

$$\alpha(q)=\alpha(-q),\ \ \lambda(q)=\lambda(-q), \tag{46}$$

the function $\alpha(q)+\lambda(q)f(\rho)$ is bounded:

$$0<\alpha(q)+\lambda(q)f(\rho)\le 1, \tag{47}$$

and the scalar field potential specified by (45) in accordance with (46) can assume values from

$$\varphi\to 0\text{ ,under }\alpha(q)+\lambda(q)f(\rho)\to 1, \tag{48}$$

to

$$\varphi\to\infty\text{ ,under }\alpha(q)+\lambda(q)f(\rho)\to 0. \tag{49}$$

In region I, in accordance with (42),(47)

$$\alpha(q)\to 1,\ \ \lambda(q)\to 0, \tag{50}$$

According to (43) the potential of the scalar field in region II tends to zero, whereas in $q\in(-a-\Delta q,-a+\Delta q)\cup(a-\Delta q,a+\Delta q)$ and any fixed value of the variable $\rho=\rho_0\in\left[0,\ R-\Delta\rho\right)\cup\left(R+\Delta\rho,\ +\infty\right)$, must be performed

$$\alpha(q)+\lambda(q)f(\rho_0)\to 1. \tag{51}$$

In region III, for the same values $q\in(-a-\Delta q,-a+\Delta q)\cup(a-\Delta q,a+\Delta q)$ and at $\rho=\rho_0\in\left(R-\Delta\rho,\ R+\Delta\rho\right)$

$$0<\alpha(q)+\lambda(q)f(\rho_0)<1. \tag{52}$$

Eqs. (50) and (51) it follows that for all $\rho\in\left[0,\ R-\Delta\rho\right)\cup\left(R+\Delta\rho,\ +\infty\right)$ the values of the function $f(\rho)$ tends to constant:

$$f(\rho)\Big|_{\rho\in\left[0,\ R-\Delta\rho\right)\cup\left(R+\Delta\rho,\ +\infty\right)}\to f_0=const. \tag{53}$$

Moreover $f_0 \neq 0$, while the functions $\alpha(q)$ and $\lambda(q)$ are interconnected:

$$\lambda(q) = \frac{1}{f_0}(1-\alpha(q)). \quad (54)$$

Substituting (54) into (52), we obtain for region III

$$0 < \alpha(q) + \left(1-\alpha(q)\right)\frac{f(\rho)}{f_0} < 1, \quad (55)$$

this together with (49), (55) mean that, at $\varphi \to \infty$

$$\alpha(q) \to 0,\ f(\rho) \to 0. \quad (56)$$

Thus, the function $\alpha(q)$ and $f(\rho)$ in the expression for the scalar field potential (45) are bounded and, for any $q \in \left(-\infty,\ +\infty\right)$ and $\rho \in \left[0, +\infty\right)$, take on values in the intervals:

$$0 < \alpha(q) < 1,\ \ 0 < f(\rho) < f_0, \quad (57)$$

moreover, according to (50):

$$\alpha(q)\Big|_{q\in\left(-\infty,-a-\Delta q\right)\cup\left(-a+\Delta q,a-\Delta q\right)\cup\left(a+\Delta q,\ +\infty\right)} \to 1, \quad (58)$$

in region I, whereas conditions (56) with regard for the symmetry of the function $\alpha(q)$ (equality (46)) yield:

$$\lim_{q\to\pm a}\alpha(q) \to 0. \quad (59)$$

The distribution for the function $f(\rho)$ at $\rho \in \left[0,\ R-\Delta\rho\right) \cup \left(R+\Delta\rho,\ +\infty\right)$ is determined by (53), according to (56)

$$f(\rho)\Big|_{\rho\to R} \to 0. \quad (60)$$

It can be shown that in region III, at $q \to \pm a$, $\rho \to R$, with (59), (60), the function $\varphi_{,q}$ and $\varphi_{,\rho}$ we obtain:

$$\varphi_{,q} = -\frac{\alpha_{,q}}{\alpha(q)}, \quad \varphi_{,\rho} = -\frac{f_{,\rho}}{f(\rho)}, \quad (61)$$

according to (41), at $\Delta q \to 0,\ \Delta\rho \to 0$

$$\left|\frac{\alpha_{,q}}{\alpha(q)}\right|\Bigg|_{q\to\pm a}\to\infty, \quad \frac{f_{,\rho}}{f(\rho)}\Bigg|_{\rho\to R}\to 0, \quad \frac{\alpha_{,q}}{\alpha(q)}\times\frac{f_{,\rho}}{f(\rho)}\Bigg|_{q\to\pm a,\ \rho\to R}\to 0 \ . \tag{62}$$

As an example, the functions $\alpha(q)$ and $f(\rho)$, satisfying the found conditions can be chosen as follows:

$$\alpha(q)=\exp\left\{-\left(\frac{1}{\varepsilon_1+(\xi_1(a+q))^2}+\frac{1}{\varepsilon_2+(\xi_2(a-q))^2}\right)\right\}, \tag{63}$$

$$f(\rho)=f_0\exp\left(-\mu\left(1-\exp\left(\frac{-1}{(\varsigma(\rho-R))^2}\right)\right)\right), \tag{64}$$

where the constants ξ_1,ξ_2 and ς determine the size ("thickness") of the rings with the scalar field concentrated inside with respect to the variables q and ρ, respectively, and positive constants $\varepsilon_1=\varepsilon_2$ and μ provide the conditions (59), (60), (62), with $q\to\pm a,\ \rho\to R,\ \Delta q\to 0,\ \Delta\rho\to 0$, namely, at

$$\begin{aligned}&\Delta q<<1,\ \ \varepsilon_1<<1,\ \varepsilon_2<<1,\\ &\Delta\rho<<1,\ \ \mu>>1,\end{aligned} \tag{65}$$

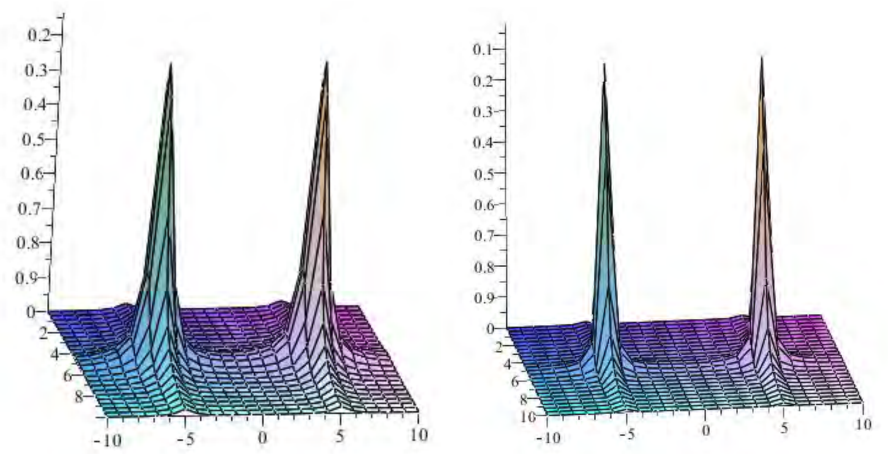
With a further contraction into a one-dimensional object (null string), i.e., at $\Delta q\to 0,\ \Delta\rho\to 0$

$$\varepsilon_1\to 0,\ \ \varepsilon_2\to 0,\ \mu\to\infty. \tag{66}$$

On Fig. 1 presents the distribution of the function $\alpha(q)+(1-\alpha(q))\left(f(\rho)/f_0\right)$, in the region $a=5,\ q\in[-10,10],\rho\in[0,10]$, for the functions $\alpha(q), f(\rho)$, specified by Eqs. (63), (64). One can see from these figures that, with increasing values of the constants ξ_1,ξ_2,ς the region of the non-unity function $\alpha(q)+(1-\alpha(q))\left(f(\rho)/f_0\right)$ (i.e., the region, where the scalar field is concentrated, and the scalar field potential isn't tend to zero) contracts, which corresponds to a decrease of the "thickness" of the rings with the scalar field concentrated inside.

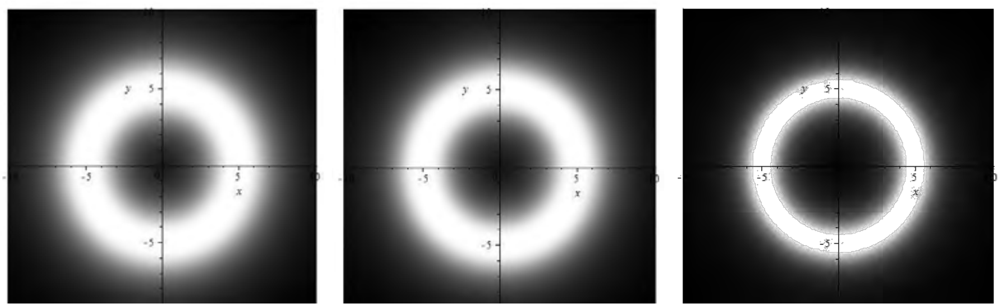
On Fig. 2, show the distribution of the scalar field potential specified by (45), (63), (64) with respect to the variable ρ ($\rho\in[0,10]$) at $R=5,\ a=5,\ \xi_1=\xi_2=1,\ \mu=4,\ \varepsilon_1=\varepsilon_2=0.01$ and $q=5.01$ with the following constants ς: a) $\varsigma=0.5$, б) $\varsigma=0.7$, c) $\varsigma=1$. Here, black color shows an area in which $\varphi\to 0$. One can see that, with increasing constant ς, the region of the non-zero scalar field potential contracts, which corresponds to a decrease of the "thickness" of the rings with the scalar field concentrated inside with respect to ρ.

On Fig. 3 one can see the distribution of the scalar field potential specified by (45) on the surface $\theta = const.$, $q \in [-10,10]$. It is obvious that an increase of the constants ς, ξ_1, ξ_2 results in the contraction of the region with the non-zero scalar field potential. In other words, the "thickness" of the rings, where the scalar field is concentrated, decreases.



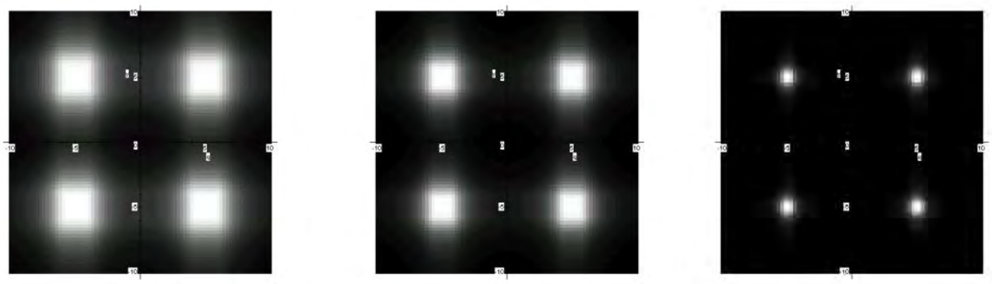
(a) (b)

Fig. 1. Distribution of the function $\alpha(q)+(1-\alpha(q))\left(f(\rho)/f_0\right)$, where $q\in\left[-10,10\right], \rho\in\left[0,10\right]$ at $R=5,\ \varepsilon_1 = \ \varepsilon_2 = 0.01$: *a)* $\mu=\xi_1=\xi_2=2$, b) $\mu=\xi_1=\xi_2=4$.



a) b) c)

Fig. 2. Scalar field potential distribution specified by (45), (63), (64) with respect to the variable ρ ($\rho\in\left[0,10\right]$) at $R=5,\ a=5,\ \xi_1=\xi_2=1,\ \mu=4,\ \varepsilon_1=\varepsilon_2=0.01$ and $q=5.01$ with the following constants ς: a) $\varsigma=0.5$, б) $\varsigma=0.7$, c) $\varsigma=1$.



a) b) c)

Fig. 3. Scalar field potential distribution specified by (45) on the surface $\theta = const.$, $q \in [-10,10]$ $\rho \in [0,10]$, which are correspond to the selection of constant values that:

a) $\varepsilon_1 = \varepsilon_2 = 0,01;$ $\xi_1 = \xi_2 = 0,5;$ μ=1; $\zeta = 0,5;$ $R = 5;$ $a = 5$

b) $\varepsilon_1 = \varepsilon_2 = 0,01;$ $\xi_1 = \xi_2 = 0,7;$ μ=1; $\zeta = 0,5;$ $R = 5$,

c) $\varepsilon_1 = \varepsilon_2 = 0,01;$ $\xi_1 = \xi_2 = 2;$ μ=1; $\zeta = 2;$ $R = 5$.

CONCLUSIONS

In this article, we have received, general view of distribution of potential scalar field for a system consisting of two "thick" null string moving without interaction along the axis in the same direction. Conditions on scalar field potential at which, within the limit of the compression component of the energy-momentum tensor of the scalar field asymptotically coincide with the components of the energy-momentum tensor for a system of two null strings moving on the same trajectory are found. The example of the potential distribution of a scalar field, corresponds to the conditions obtained.

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Леляков О. П. Розподіл потенціалу скалярного поля для системи з двох замкнутих нуль-струн незмінного з часом радіуса, що рухаються уздовж осі z / О. П. Леляков, А. О. Ковальов // Вчені записки Таврійського національного університету імені В. І. Вернадського. Серія : Фізико-математичні науки. – 2014 – Т. 27 (66), № 2. – С. 37-49.

У роботі запропоновано загальний вигляд розподілу потенціалу дійсного безмассового скалярного поля для «розмазаної» нуль-струни, що радіально розширюється в площині $z = 0$. Знайдено умови на потенціал скалярного поля, при яких, в межі стиснення в одновимірний об'єкт, компоненти тензора енергії-імпульсу скалярного поля асимптотично збігаються з компонентами тензора енергії-імпульсу замкнутої нуль-струни, що рухається по тій же траєкторії.

Ключові слова: нуль-струна, скалярне поле, космологія.

Леляков А. П. Распределение потенциала скалярного поля для системы из двух замкнутых нуль-струн неизменного со временем радиуса, движущихся вдоль оси z / А. П. Леляков, А. О. Ковалев // Ученые записки Таврического национального университета имени В. И. Вернадского. Серия : Физико-математические науки. – 2014. – Т. 27 (66), № 2. – С. 37-49.
В работе предложен общий вид распределения потенциала вещественного безмассового скалярного поля для «размазанной» нуль-струны радиально расширяющейся в плоскости $z = 0$. Найдены условия на потенциал скалярного поля, при которых, в пределе сжатия в одномерный объект, компоненты тензора энергии-импульса скалярного поля асимптотически совпадают с компонентами тензора энергии-импульса замкнутой нуль-струны движущейся по той же траектории.
Ключевые слова: нуль-струна, скалярное поле, космология.

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