Scientific Notes of Taurida National V. I. Vernadsky University
Series : Physics and Mathematics Sciences. Volume 27 (66). 2014. No. 2. P. 14-22

UDK 535.42

ALGEBRA OF OPTICAL QUARKS

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We have considered a new type of singular beams called as optical quarks. They have fractional topological charges being equal to half an integer and they possess rather unique properties. There are four types of optical quarks, even and odd ones, which reveal the opposite signs of topological charges. The sums or differences of the even and odd quarks form standard vortex or non-vortex beams with the topological charges of integer order. All the quarks in the same beam annihilate and the beam vanishes. We conducted an analysis of all possible combinations of even and odd optical quarks with different charges. What provided an opportunity to explore what interactions correspond to their "sum" and "difference".

Keywords: optical vortex, fractional charge.

PACS: 42.25.Fx, 47.32.C

INTRODUCTION

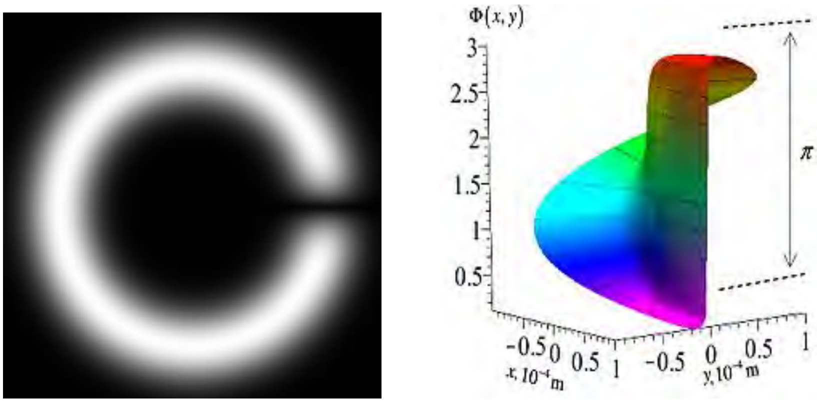
As far back as in the beginning of 1990s, Soskin et al. [1] have wondered at a 'strange' behaviour of the simplest singular beams with fractional topological charges. It turns out that the inherent property of such beams is that the initial field distribution is not recovered during propagation along any beam length, while the optical vortex with a fractional topological charge is not nucleated at any beam cross section. Indeed, a broken axial symmetry of the beam does not permit reconstructing the propagating field. The immediate inference is that the vortex beams with the fractional topological charges cannot exist in principle. Although such a simple statement does not need a strong confirmation at all, the work by Berry [2] has ignited a heated discussion. Berry has considered the process of diffraction for a Gaussian (G) beam by a spiral phase plate with the fractional phase step. The evolution of the diffracted beam manifests itself in the form of beam fracture, with chains of singly charged optical vortices. However, the major point has been that the beam could carry over a fractional orbital angular momentum (OAM).

An avalanche of subsequent studies has surpassed all imagination [4]. The detailed analysis has shown that the fractional optical vortex splits into an infinite series of integer-order vortices, while the OAM of the beam is defined by contributions of integer-order optical vortices. Although it seems at the first sight that fractionalising the OAM of the beam contradicts the foundations of quantum mechanics, the authors of the work [7] have shown the mixed stated of photons to be able to carry over the fractional OAMs. At the same time, according to the results [2], the fractional-vortex beam must inevitably be destroyed while propagating, because of different phase velocities of partial elementary beams involved. Nevertheless, the recent studies [3,6] have demonstrated availability of spatially invariant beams with fractional OAMs and, in particular, with fractional optical vortices [3,6] (so-called Erf-G beams).

Such unusual properties of the fractional-vortex beams compel to peer more attentively into the structure of spatially invariant fractional-vortex beams. The aim of the present article is to analyse the interaction features of elementary fractional-vortex beams in the free space.

1. THEORY

We have shown in the study [3,6] that the error function-Gaussian beams (Erf-G beams) bearing optical vortices with the topological charges $l = +1/2$ (Fig.1) belong to the strong solutions of the vector paraxial wave equation and refer to a set of so-called standard paraxial beams (Hermite-Gaussian (HG), Laguerre-Gaussian (LG), Bessel-Gaussian (BG), etc.), with a complex argument.



(a) (b)

Fig. 1. Intensity (a) and phase (b) distributions for erf- beams with $w_0 = 35\ \mu$m in the initial plane $z = 0$ (a) m^{-1} and (b) m^{-1}.

In contrast to the usual standard beams (e.g., HG, Laguerre (L), G or BG ones), the erf-G beams have a non-factorising form, i.e. their azimuthal (p) and radial (r) variables are not separated. The scalar erf-G beam may be written in the form

$$\psi_s = \frac{-2i\sqrt{\pi}\, e^{\frac{is\varphi}{2}}}{R} NG\left\{ e^{\frac{-R^2}{2}} erf\left(iR\sin\frac{\varphi}{2} \right) + s\, e^{\frac{R^2}{2}} erf\left(R\cos\frac{\varphi}{2} \right) \right\}, \qquad (1)$$

where $erf(x)$ stands for the error function, $R=\sqrt{2\frac{Kr}{\sigma}}$, $G=\frac{\exp\left(\frac{-r^2}{W_0^2\sigma}\right)}{\sigma}$, $N=\exp\left(\frac{-K^2w_0^2}{4\sigma}\right)$, $\sigma = 1 - i\cdot z/z_0$, and $z=-kw_0^2/2$. Here w_0 is the radius of the beam waist at $z = 0$, k the wavenumber, $s = \pm1$, and the free parameter K can acquire arbitrary values, including complex ones. The field distribution given by Eq. (1) depends on the free parameter K.

Near the beam axis where Kr is very small ($Kr << 1$), the wave function of the erf-G beam given by Eq. (1) may be presented as

$$\psi_s \approx 2\sqrt{\pi}e^{\frac{is\varphi}{2}}NG\left\{e^{\frac{-R^2}{2}}\sin\frac{\varphi}{2}-ise^{\frac{R^2}{2}}\left(R\cos\frac{\varphi}{2}\right)\right\}, \tag{2}$$

Notice that the two terms in Eq. (2) have much to do with the forms of the fractional-vortex beams suggested by Soskin et al. in the work [1]. Let us write out a generalised form of such wave constructions at the initial plane $z = 0$ and outline their basic properties:

$$Q_{ev}^{+,m}=\cos\left(\frac{m\varphi}{2}\right)e^{\frac{im\varphi}{2}}F(r), \tag{3}$$

$$Q_{od}^{+,m}=-i\sin\left(\frac{m\varphi}{2}\right)e^{\frac{im\varphi}{2}}F(r), \tag{4}$$

$$Q_{ev}^{-,m}=-\cos\left(\frac{m\varphi}{2}\right)e^{\frac{-im\varphi}{2}}F(r), \tag{5}$$

$$Q_{od}^{-,m}=-i\sin\left(\frac{m\varphi}{2}\right)e^{\frac{-im\varphi}{2}}F(r), \tag{6}$$

where $F(r)$ is the radial envelope of the standard paraxial beam and $m = 2m'+1$ is an odd number.

An arbitrary non-vortex beam can be presented as a superposition of the wave elements given by Eqs. (3)-(6):

$$F(r)=Q_{ev}^{+,m}+Q_{od}^{+,m} \quad or \quad F(r)=-\left(Q_{ev}^{-,m}+Q_{od}^{-,m}\right) \tag{7}$$

Correspondingly, the beams with the edge dislocations may be written as

$$F(r)\cos(m\varphi)=\left\{Q_{ev}^{+,m}-Q_{od}^{+,m}-Q_{ev}^{-,m}+Q_{od}^{-,m}\right\} \tag{8}$$

$$F(r)\cos(m\varphi)=\left\{Q_{ev}^{+,m}-Q_{od}^{+,m}+Q_{ev}^{-,m}-Q_{od}^{-,m}\right\} \tag{9}$$

However, since the vortex beams of high orders are unstable with respect to slight perturbations, later on we will focus our attention only on the simplest vortex beams with $m=\pm 1$.

Another basic property is that the sum all the wave constructions given by Eqs. (3)-(6) vanishes:

$$Q_{ev}^{+,m}+Q_{od}^{+,m}+Q_{ev}^{-,m}+Q_{od}^{-,m}=0\,. \tag{10}$$

In analogy with the Gell-Mann quark model of hadrons, we will call such wave constructions *as* optical quarks. In this way the wave constructions $Q_{od}^{-,m}$ *and* $Q_{ev}^{-,m}$ may be treated as anti-quarks. Eqs. (7) and (8) can be read such that a superposition of two even and odd quarks or anti-quarks forms a non-vortex beam, while their difference represents a vortex-beam. At the same time, the occurrence of all the quarks and anti-quarks results in their total annihilation (see Eq. (10)).

2. GENERATION OF ERF-G BEAMS

Generation of optical quarks is possible due to the use of computer-generated holograms [1] shown in Fig. 2.

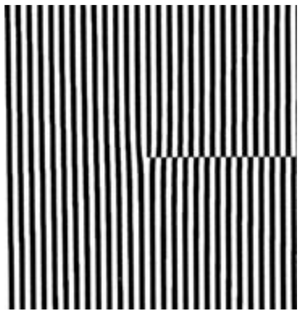


Fig. 2. Computer generated hologram.

For the generation of optical quarks and their analysis was assembled Mach-Zehnder interferometer (Fig. 3). For the light source we used a He-Ne laser LGN-207A. After the laser light propagated by splitting cube, where the second beam was directed into the support arm of the interferometer. The main beam spread and entered to polarizer. There he converted into a linearly polarized, and then fell by a quarter wave plate, and converted into a circularly polarized. After that, the beam was diffracted by the hologram (Fig. 3(6)). On the phase transporant generated a spectrum of beams (Fig. 4). Central order is a beam of zero charge, each next beam differs from the preceding one by l = 1/2 in dependence of order. Thus for the experiment, we are only suitable −1 and +1 order.

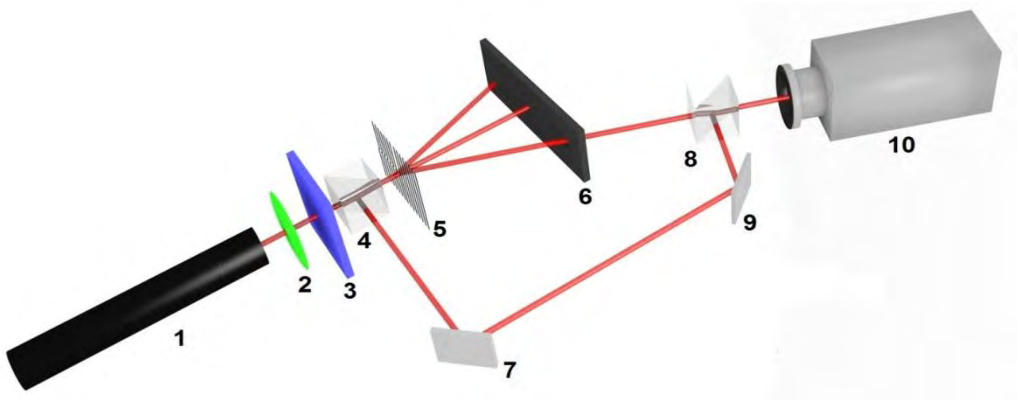


Fig. 3. The experimental setup for the generation of fractional topological charges: 1 − He-Ne laser; 2 – polarizer; 3 − quarter wave plate; 4, 8 − splitting cube; 5 – hologram; 6 – diaphragm; 7, 9 – mirror; 10 − camera.

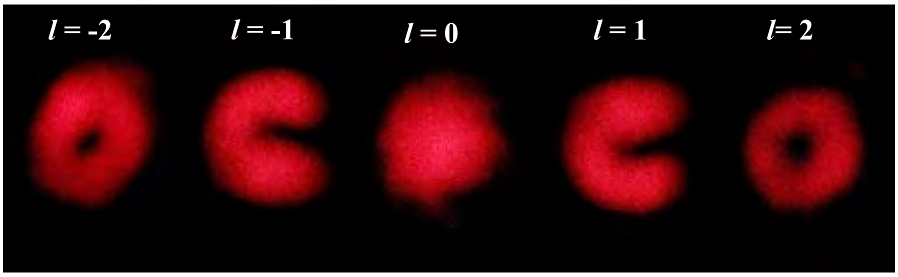
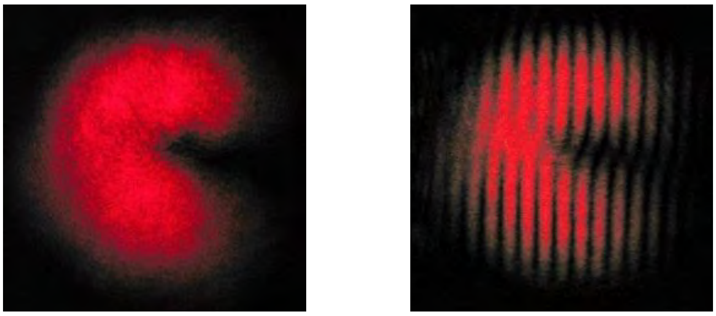


Fig. 4. The spectrum of beams carrying fractional charge.



(a) (b)

Fig. 5. Intensity (a) and phase (b) distributions for erf- beams with $l = 1/2$.

From the spectrum of the hologram we chose the needed optical quarks (+1 and −1 order) Fig. 5. Final beam interfered with the reference beam (Fig. 5, b). Overall picture of falling on the CCD camera.

3. THE ALGEBRA OF FRACTIONAL CHARGES

To study the interaction of fractional charges has been assembled experimental set-up (Fig. 6). For the light source used laser LGN-207B (1), a power of 0.5 mW, $\lambda = 632$ microns. When linearly polarized light passes through the quarter-wave plate (3), the beam is converted into a circularly polarized, and placed in a Mach-Zehnder interferometer. Consisting of splitting the cubes (4, 8) and mirrors (7). Each of the arms in the direction of propagation of the beam was placed computer synthesized hologram (5).

After output generates eddies fractional charges with different combinations of ($l = 1/2$; $l = +1/2$), ($l = +1/2$, $l = -1/2$), ($l = -1/2$, $l = -1/2$), ($l = -1/2$, $l = +1/2$). We took one of these combinations. Both vortices were combined by splitting cube (5), after which there was the result of their interaction. The result of the experiment were recorded by CCD camera (9) and displayed on the monitor.

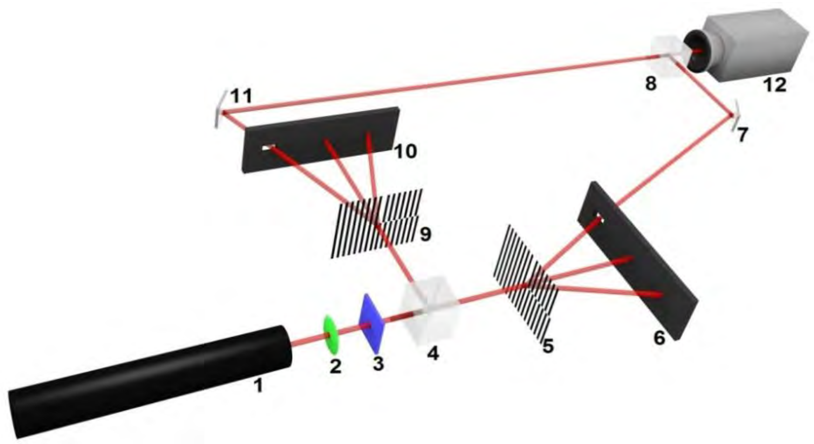
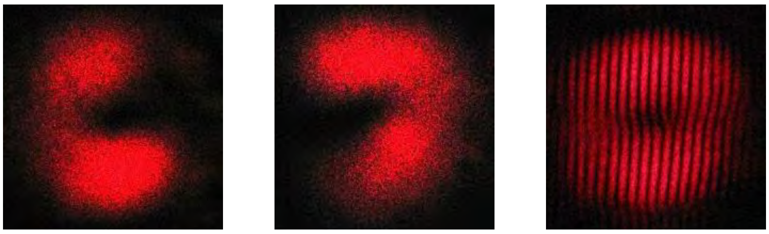


Fig. 6. The experimental set-up for the "addition" and "subtraction" fractional topological charges: 1 – HeNe laser; 2 – polarizer; 3 – quarter wave plate; 4, 8 – splitting cubes; 5, 9 – hologram; 6, 10 – diaphragm; 7, 11 – mirror; 12 – camera.

Since there are four combinations of addition of beam intensity, we need to choose the most interesting. As shown by theoretical calculations combination ($l = 1/2$, $l = -1/2$) and the combination of ($l = -1/2$, $l = +1/2$) lead to the same type of addition of beam intensity. A combination of ($l = 1/2$, $l = +1/2$) and ($l = -1/2$, $l = -1/2$) are identical but opposite topological charges.

4. THE ADDITION OF OPTICAL VORTICES WITH FRACTIONAL CHARGE

During the experiment it was shown that when placed in the assembled installation scheme of hologram ($l = 1/2$, $l = +1/2$) is generated at the output of the interference pattern (Fig. 7) is the sum formed by the addition of the intensity of the beams.



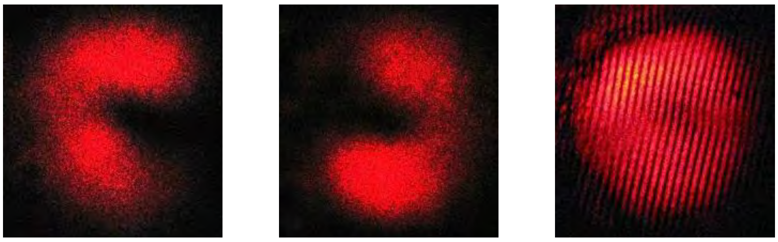
(a) (b) (c)

Fig. 7. The addition of optical vortices with fractional charge: (a) optical vortex of charge $l = 1/2$; (b) optical vortex of charge $l = 1/2$; (c) the resulting interference pattern.

But since each of these beams has its own topological charge $l = 1/2$, the result of addition of these beams is formed interference pattern displayed on it with the topological sum of two optical vortices. As can be seen from the interference pattern of the resulting beam is equal to the topological charge $l = 1$. Thus is explained the law of conservation of topological charges.

5. SUBTRACTION OF OPTICAL VORTICES WITH FRACTIONAL CHARGE

During the experiment it was shown that when placed in the assembled installation scheme hologram ($l = 1/2$, $l = -1/2$), the output interference pattern generated, which is the sum formed by adding the intensities of the beams (Fig. 8). But since each of these beams has its own topological charge $l = 1/2$, the result of addition of these beams is formed interference pattern displayed on it with the topological sum of two optical vortices.



(a) (b) (c)

Fig. 8. The subtraction of optical vortices with fractional charge: (a) optical vortex of charge $l = 1/2$; (b) optical vortex of charge $l = -1/2$; (c) the resulting interference pattern.

As can be seen from the resulting interference pattern has a beam topological charge equal to $l = 0$ (Fig. 8, c). Thus is explained the law of conservation of topological charges.

CONCLUSION

So, a new type of singular beams called as optical quarks was considered. They have fractional topological charges being equal to half an integer and they possess rather unique

properties. There are four types of optical quarks, even and odd ones, which reveal the opposite signs of topological charges. The sums or differences of the even and odd quarks form standard vortex or non-vortex beams with the topological charges of integer order. All the quarks in the same beam annihilate and the beam vanishes. All possible combinations of even and odd optical quarks with different charges were analized. What provided an opportunity to explore what interactions correspond to their "sum" and "difference".

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Егоров Ю. О. Алгебра оптических кварков / Ю. О. Егоров, В. Л. Коноваленко, О. В. Воляр // Вчені записки Таврійського національного університету імені В. І. Вернадського. Серія : Фізико-математичні науки. – 2014. – Т. 27 (66), № 2. – С. 14-22.
Було розглянуто новий тип сингулярних пучків, так званих оптичних кварків. Дані сингулярні пучки володіють топологічним зарядом рівним половині цілого числа, а також вони мають досить унікальні оптичні властивости. Всього існує чотири типи оптичних кварків: парні і непарні, що мають різні знаки топологічних зарядів. Додавання і віднімання парних і непарних оптичних кварків призводить до створення стандартного пучка з топологічним зарядом рівному цілому числу. Був проведений аналіз всіх можливих комбінацій парних і непарних оптичних кварків з різними зарядами, що дало можливість досліджувати взаємодії оптичних кварків як при підсумовуванні, так і відніманні.
Ключові слова: оптичний вихор, дробовий топологічний заряд.

Егоров Ю. А. Алгебра оптических кварков / Ю. А. Егоров, В. Л. Коноваленко, А. В. Воляр // Ученые записки Таврического национального университета имени В. И. Вернадского. Серия : Физико-математические науки. – 2014. – Т. 27 (66), № 2. – С. 14-22.
Был рассмотрен новый тип сингулярных пучков, называемых «оптические кварки». Данные сингулярные пучки обладают топологическим зарядом равным половине целого числа, также они обладают достаточно уникальными оптическими свойствами. Всего существует четыре типа оптических кварков: четные и нечетные, имеющие разные знаки топологических зарядов. Сложение и вычитание четных и нечетных оптических кварков приводит к созданию стандартного пучка с топологическим зарядом равному целому числу. Был проведен анализ всех возможных комбинаций четных и нечетных оптических кварков с разными зарядами, что дало возможность исследовать взаимодействия оптических кварков при суммировании и при их вычитании.
Ключевые слова: оптический вихрь, дробный топологический заряд.

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Received 19 September 2014.