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LOCALIZATION OF FUNDAMENTAL MODES ON A TWIST DEFECT IN A TWISTED ANISOTROPIC FIBER

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In this paper we have studied the influence of a twist defect on propagation of a circularly polarized Gaussian mode in a Bragg twisted weakly guiding fiber with a uniaxial material anisotropy. It has been shown that right circularly polarized Gaussian mode in a narrow spectral range inside of forbidden band almost completely passes the fiber. This process is accompanied by generation of a localized on the defect fundamental modes.
Keywords: twisted anisotropic fiber, twist defect, fundamental mode.

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INTRODUCTION

Interest in optical features of a non-guiding medium with a rotating axis of linear anisotropy have appeared long ago. As early as 1944 V. L. Ginzburg studied propagation of electromagnetic field in a gyrotropic medium and found out that in the general case the field in this media has elliptical polarization in coordinate system that traces local orientation of linear anisotropy's axis. Later on a mathematical formalism for description of twisted anisotropic fibers was developed for both regular and irregular twist [1-7]. As a result of those studies it has been established that twisting the anisotropic fiber allows one to reduce polarization mode dispersion (PMD) for nonsingular beams. In recent papers it has been shown that an analogous effect takes place also for singular beams [8-10]. Nevertheless, the question of propagation of electromagnetic field in such fibers with a twist defect is still open. In some cases violation of fiber's rotational symmetry with a special transverse profile can lead to conversion of fundamental mode to an optical vortex [11]. In the present paper we will show that the violation of rotational symmetry, which manifests in breaking the twist periodicity of the anisotropic fiber, can lead to undamped passage of an incoming regular beam in a narrow spectral range within the forbidden spectral band and generation of the defect mode localized on the twist defect.

1. MODEL AND BASIC EQUATION

Consider twisted anisotropic fiber (TAF) (Fig. 1), in which the transverse part of the tensor of refractive index in a cross-section with coordinate z is described as

$$\hat{n}_t^2(z) = \bar{n}^2(1-2\Delta f)\hat{1} + \Delta n^2 \begin{pmatrix} 0 & e^{-2iqz} \\ e^{2iqz} & 0 \end{pmatrix}, \qquad (1)$$

where $\overline{n}^2 = \left(n_e^2 + n_o^2\right)/2, \Delta n^2 = \left(n_e^2 - n_o^2\right)/2$, $n_{e,o}$ – extraordinary and ordinary refractive indexes, Δ – optical contrast between fiber's cladding and core, f – profile function, $q = 2\pi / H$, H – twist pitch of the anisotropic fiber.

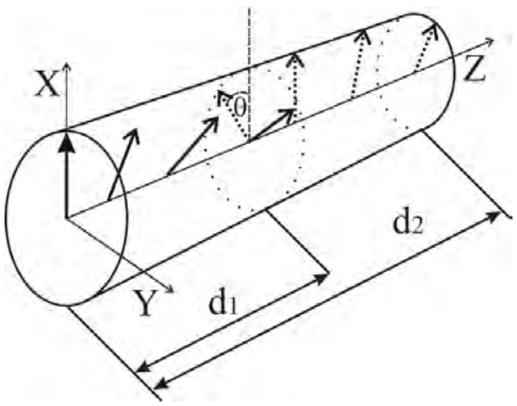


Fig. 1. The model of twisted anisotropic fiber with a twist defect. The defect is characterized by the angle θ between the initial direction of anisotropy (solid arrows) axis and the one of anisotropy axis at length d_1. The total length of the fiber is d_2, $\left(X, Y, Z\right)$ – laboratory coordinate system.

Upon substitution of expression (1) into the Maxwell equations it turns out that spatial-temporal evolution of transverse component of electromagnetic field essentially depends on longitudinal component of the field [8]. Nevertheless, at condition $\Delta n^2 \ll \Delta \ll 1$ one can neglect the influence of the longitudinal component on the transverse component and write down the equation in transverse components in the basis of circular polarization as:

$$\left(\nabla^2 + k^2 \overline{n}^2\left(x, y\right) + \frac{1}{2} k^2 \Delta n^2 \begin{pmatrix} 0 & e^{-2iqz} \\ e^{2iqz} & 0 \end{pmatrix}\right)\left|\psi\right\rangle = 0. \qquad (2)$$

where $k = 2\pi / \lambda$ – wavenumber, λ is wavelength in vacuum, $\left|\psi\right\rangle = \mathrm{col}\left(E_+, E_-\right)$, $E_\pm = (E_x \mp iE_y)/\sqrt{2}$. It should be noted that in Eq. (2) the influence of the spin-orbit interaction is neglected.

An important feature of Eq. (2) is the absence of translation invariance along the fiber's axis. To restore it one has to make standard substitution $\tilde{E}_\pm = E_\pm e^{\pm iqz}$ and replace variables $r = r'$, $z = z'$, $\varphi - qz = \varphi'$, where cylindrical polar coordinates $\left(r, \varphi, z\right)$ are implied. After these substitutions Eq. (2) takes the form [8]:

$$\left\{\hat{H}_0 - q^2\hat{\sigma}_0 + 2q\beta\hat{\sigma}_3 + \frac{1}{2}k^2\Delta n^2\hat{\sigma}_1\right\}\left|\tilde{\psi}\right\rangle = \beta^2\left|\tilde{\psi}\right\rangle, \tag{3}$$

where $\left|\tilde{\psi}\right\rangle = \mathrm{col}\left(\tilde{E}_+, \tilde{E}_-\right)$, $\hat{H}_0 = \nabla_t^2 + \bar{n}^2\left(1 - 2\Delta f\right)\hat{1}$.

2. *l* = 0 MODES OF A TWISTED ANISOTROPIC FIBER

To solve Eq. (3) one can use perturbation theory. As the spectrum of $\hat{H}_0$ is twofold degenerate, it is necessary to use perturbation theory with degeneracy. After a little algebra one can obtain a 2×2 perturbation matrix in an $\hat{H}_0$-representation for $l = 0$, where l is the azimuthal number:

$$H = \begin{pmatrix} \tilde{\beta}_0^2 - \left(\beta - q\right)^2 & Q \\ Q & \tilde{\beta}_0^2 - \left(\beta + q\right)^2 \end{pmatrix}, \tag{4}$$

where $Q = 0.5\left(k\Delta n\right)^2$ and $\tilde{\beta}_0$ – the scalar propagation constant. It should be noted that matrix (4) depends on its eigenvalues. Far from the point $a\left(q_0, 0\right)$, where $q_0 = \tilde{\beta}_0$, one can neglect the coupling between the orthogonally polarized fields and assume that the modes of TAF is represented by four fundamental modes:

$$\left|\tilde{\psi}_{1,2}\right\rangle = \left|\pm\right\rangle \exp\left(i\tilde{\beta}_0 z\right), \left|\tilde{\psi}_{3,4}\right\rangle = \left|\pm\right\rangle \exp\left(-i\tilde{\beta}_0 z\right), \tag{5}$$

where ket-vector $\left|\pm\right\rangle$ in the basis of linear polarizations (L) has the form $\begin{pmatrix} 1 \\ \pm i \end{pmatrix}_L$. It should be noted that here the functions describing the radial distribution of the modes are omitted.

Near the point a the elements of (4) on the main diagonal tend to zero and the fields $\left|\tilde{\psi}_i\right\rangle$ undergo a strong hybridization. Using the substitution $q = q_0 + \varepsilon$, $\beta = \delta$, where $\varepsilon, \delta \ll \tilde{\beta}_0$, one can the linearize matrix elements in Eq. (4). It is straightforward to obtain the expression for H:

$$H = \begin{pmatrix} \delta - \varepsilon & Q/2\tilde{\beta}_0 \\ Q/2\tilde{\beta}_0 & -\delta - \varepsilon \end{pmatrix}. \tag{6}$$

The eigenvalues of (6) have the form:

$$\delta_{1,2} = \pm\sqrt{\varepsilon^2 - A^2} \equiv \pm R, \tag{7}$$

where $A = Q/2\tilde{\beta}_0$. The fiber's modes corresponding to (7) can be written in the laboratory frame as

$$\left|\psi_1\right\rangle=\left\{\frac{\varepsilon+R}{A}\left|+\right\rangle e^{-iqz}+\left|-\right\rangle e^{iqz}\right\}e^{izR},\ \left|\psi_2\right\rangle=\left\{\frac{\varepsilon-R}{A}\left|+\right\rangle e^{-iqz}+\left|-\right\rangle e^{iqz}\right\}e^{-izR}. \quad (8)$$

Obviously, the fields (8) are not normalized. The other two modes remain not hybridized.

$$\left|\psi_3\right\rangle=\left|+\right\rangle e^{i\tilde{\beta}_0 z},\ \left|\psi_4\right\rangle=\left|-\right\rangle e^{-i\tilde{\beta}_0 z}. \quad (9)$$

Making the substitution $\left|\pm\right\rangle\to\left|\pm\right\rangle\exp\left(\mp i\theta\right)$ allows one to obtain the expressions for the modes in the laboratory frame in the second part of the fiber (that is beyond the defect), which is rotated through an angle θ with respect to the first one.

3. GENERATION OF LOCALIZED STATES IN A TWISTED ANISOTROPIC FIBER

To study the evolution of the Gaussian beam with a left circular polarization in a TAF with the twist defect one has to expand the incoming field in the eigenmodes of the fiber. Then the incoming Gaussian beam with a left circular polarization, which can be approximated as $\left|-\right\rangle$, on the left of the fiber $\left(z\le 0\right)$ can be written as

$$\left|F_{in}\right\rangle=\left|-\right\rangle\exp\left(ikz\right)+\left(X_1\left|+\right\rangle+X_2\left|-\right\rangle\right)\exp\left(-ikz\right). \quad (10)$$

In this expression one takes into account that in the reflected light there may appear the fields with orthogonal polarizations. Within the TAF one can write

$$\left|F_1\right\rangle=X_3\left|\psi_1\right\rangle+X_4\left|\psi_2\right\rangle+X_5\left|+\right\rangle\exp\left(i\tilde{\beta}_0 z\right)+X_6\left|-\right\rangle\exp\left(-i\tilde{\beta}_0 z\right) \quad (11)$$

at $z\in\left[0,d_1\right]$, and

$$\left|F_2\right\rangle=X_7\left|\psi_1'\right\rangle+X_8\left|\psi_2'\right\rangle+X_9\left|+\right\rangle\exp\left(i\tilde{\beta}_0 z\right)+X_{10}\left|-\right\rangle\exp\left(-i\tilde{\beta}_0 z\right) \quad (12)$$

at $z\in\left[d_1,d_2\right]$.

On the right of the fiber the field is described as

$$\left|F_{out}\right\rangle=\left(X_{11}\left|+\right\rangle+X_{12}\left|-\right\rangle\right)\exp\left(ik\left(z-d_2\right)\right). \quad (13)$$

It is necessary to take into account that the expressions for $\left|\psi_i'\right\rangle$ in the second part can be derived with a help of the substitution $\left|\pm\right\rangle\to\left|\pm\right\rangle\exp\left(\mp i\theta\right)$ in $\left|\psi_i\right\rangle$. The unknown coefficients X_i can be found from the system obtained through matching the expressions for fields and their derivatives at the boundaries.

Consider the case where the defect is maximal $\left(\theta=\pi/2\right)$ and is located right at the middle of the TAF $\left(d_1=0.5d_2\right)$. Numerical solution of equations in the unknown coefficients X_i allows one to establish the presence of the crossover event for the reflected fundamental mode $|+\rangle$ and the transmitted fundamental mode $|-\rangle$ (Fig.2). The crossover takes place at $d = d_{co} = 13967H \approx 0.005894\ m$.

Before the crossover $(d < d_{co})$ a significant part of the energy of the incoming fundamental mode $|-\rangle$ in a narrow spectral range within the forbidden band, which boundaries can be found from the equation $|\varepsilon^2 - A^2| = 0$, passes through the TAF with a twist defect (Fig. 3, b). Simultaneously, on the graph for the reflection coefficient of the fundamental mode $|+\rangle$ a narrow dip appears (Fig. 3, a). As the length of the fiber increases, the behavior of the transmission/reflection coefficients changes. After the crossover (at $d = 2d_{co}$) a sharp peak in the transmission curve for $|-\rangle$ field and a dip in the reflection curve for $|+\rangle$ field disappear (Fig. 4).

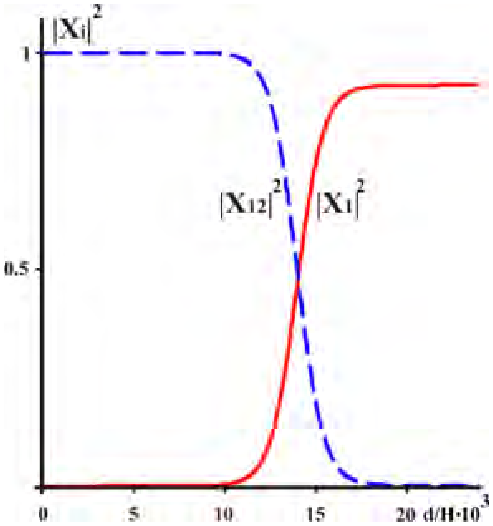


Fig. 2. Dependence of reflection coefficient for $|+\rangle$ $\left(|X_1|^2\right)$ and transmission coefficient for $|-\rangle$ $\left(|X_{12}|^2\right)$ on the reduced length d/H. Fiber parameters: $n_{co} = 1.5,\ \Delta = 10^{-2}, \Delta n^2 = 10^{-3}$, $\lambda = \lambda_0$, $\lambda_0 = 632.8\ nm$, $r_0 = 10\lambda_0$, $H = 4.21985 \cdot 10^{-7}\ m,\ \theta = \pi/2$. The incoming field is $|-\rangle$.

Appearance of a sharp pick in the middle of the forbidden band $(\lambda = 632.8\ nm)$ for the fundamental mode $|-\rangle$ at $d = d_{co}$ is accompanied by the appearance of a localized defect state. To illustrate this let us plot a graph for intensity within the TAF (Fig. 5) at $d = d_{co}$ for the fields

$$|+\rangle_{in} = \left[X_3 \frac{\varepsilon + R}{A} \exp(iRz) + X_4 \frac{\varepsilon - R}{A} \exp(-iRz) \right] \exp(-iqz) + X_5 \exp\left(i\tilde{\beta}_0 z\right),$$

$$|-\rangle_{in} = \left[X_3 \exp(iRz) + X_4 \exp(-iRz) \right] \exp(iqz) + X_6 \exp\left(-i\tilde{\beta}_0 z\right) \tag{14}$$

at $z \in [0, 0.5d_{co}]$, and

$$|+\rangle'_{in} = \left[X_7 \frac{\varepsilon + R}{A} \exp(iRz) + X_8 \frac{\varepsilon - R}{A} \exp(-iRz) \right] \exp\left(-i\left(\theta + qz\right)\right) + X_9 \exp\left(i\tilde{\beta}_0 z\right),$$

$$|-\rangle'_{in} = \left[X_7 \exp(iRz) + X_8 \exp(-iRz) \right] \exp\left(i\left(\theta + qz\right)\right) + X_{10} \exp\left(-i\tilde{\beta}_0 z\right), \tag{15}$$

at $z \in [0.5d_{co}, d_{co}]$.

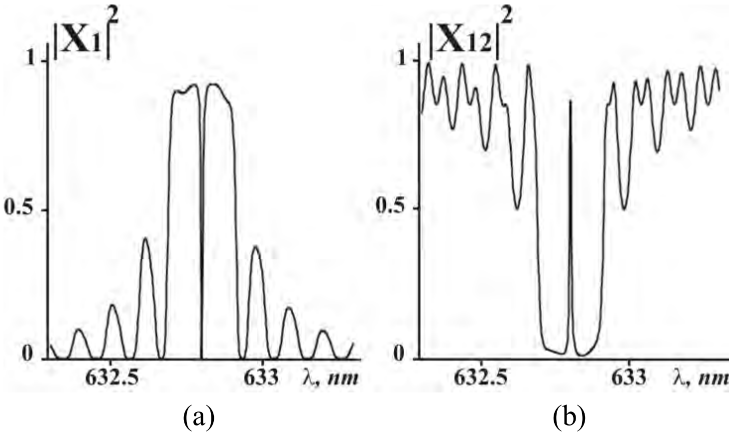
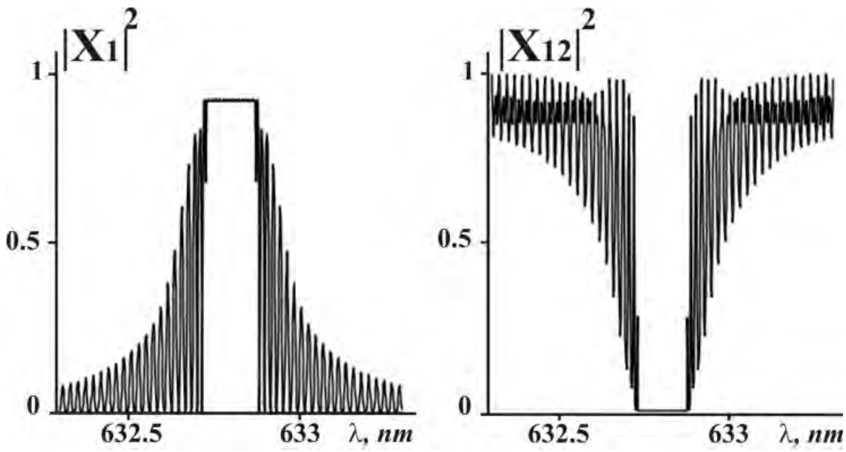


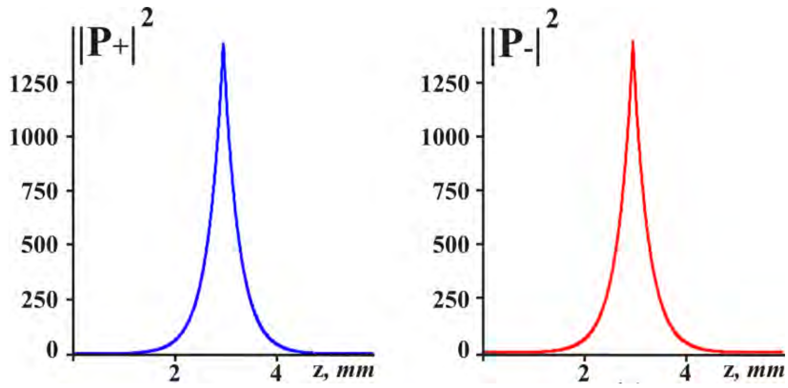
Fig. 3. Dependence of reflection coefficient for $|+\rangle$ (a) and transmission coefficient for $|-\rangle$ (b) on the wavelength of the incoming $|-\rangle$ field. Fiber parameters: $n_{co} = 1.5$, $\Delta = 10^{-2}$, $\Delta n^2 = 10^{-3}$, $\lambda = \lambda_0$, $\lambda_0 = 632.8\ nm$, $r_0 = 10\lambda_0$, $H = 4.21985 \cdot 10^{-7}\ m$, $d = 0.4d_{co}$, $\theta = \pi / 2$.

From Fig. 5 one can see that the amplitude of the fields $|+\rangle$ and $|-\rangle$ at $d = d_{co}$ significantly exceed the amplitude of the fields outside the TAF. As the length increases, the field's intensity within the fiber decreases and at $d = 2d_{co}$ the localization of the fundamental modes disappears (Fig. 6).



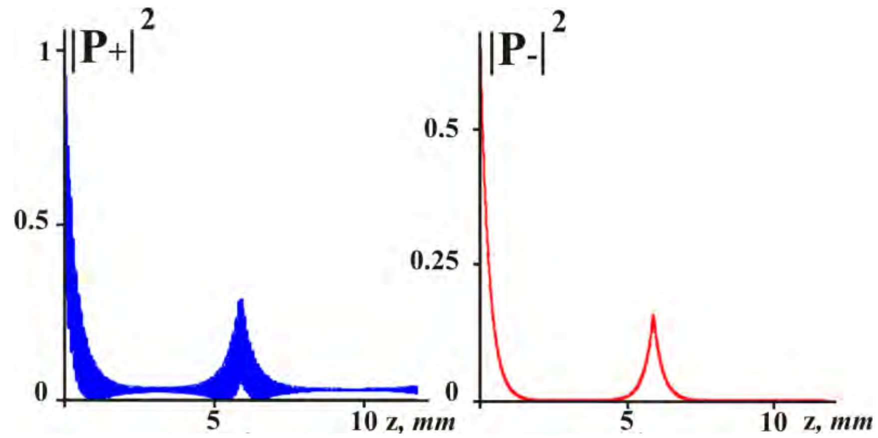
(a) (b)

Fig. 4. Dependence of reflection coefficient for $|+\rangle$ (a) and transmission coefficient for $|-\rangle$ (b) on the wavelength of the incoming field $|-\rangle$. Fiber parameters: $n_{co}=1.5$, $\Delta=10^{-2}$, $\Delta n^2=10^{-3}$, $\lambda=\lambda_0$, $\lambda_0=632.8\ nm$, $r_0=10\lambda_0$, $H=4.21985\cdot10^{-7}\ m$, $d=2d_{co}$, $\theta=\pi/2$.



(a) (b)

Fig. 5. Dependence of intensity for the fields $|+\rangle$ (a) and $|-\rangle$ (b) within the fiber on z at the incoming fundamental mode $|-\rangle$. Fiber parameters: $n_{co}=1.5, \Delta=10^{-2}, \Delta n^2=10^{-3}$, $r_0=10\lambda_0$, $\lambda_0=632.8\ nm$, $H=4.21985\cdot10^{-7}\ m$, $d=d_{co}$, $\theta=\pi/2$.



(a) (b)

Fig. 6. Dependence of intensity for fields $|+\rangle$ (a) and $|-\rangle$ (b) within the fiber on z at incoming fundamental mode $|-\rangle$. Fiber parameters: $n_{co} = 1.5, \Delta = 10^{-2}, \Delta n^2 = 10^{-3}$, $r_0 = 10\lambda_0$, $\lambda_0 = 632.8\ nm$, $H = 4.21985 \cdot 10^{-7}\ m$, $d = 2d_{co}$, $\theta = \pi/2$.

CONCLUSION

In this paper we have established that the passage of the fundamental mode $|-\rangle$ through the twisted anisotropic fiber with a twist defect in a narrow spectral range within the forbidden band can be accompanied by the appearance of the localized states $|+\rangle$ and $|-\rangle$ within the fiber. The amplitude of the localized state is maximal right at the twist defect.

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Алєксєєв К. М. Локалізація фундаментальних мод на дефекті скрутки в скрученому анізотропному волокні / К. М. Алєксєєв, Б. П. Лапін, М. А. Яворський // Вчені записки Таврійського національного університету імені В. І. Вернадського. Серія : Фізико-математичні науки. – 2014. – Т. 27 (66), № 2. – С. 5-13.
У даній статті вивчено вплив дефекту скрутки на поширення циркулярно поляризованої гаусової моди в брегґівському слабкоспрямовуючему скрученому волокні з одноосьової матеріальної анізотропією. Було показано, що гаусова мода з правої круговою поляризацією у вузькому спектральному діапазоні в межах забороненої зони майже повністю проходить крізь волокно. Даний процес супроводжується появою локалізованих на дефекті фундаментальних мод.
Ключові слова: скручене анізотропне волокно, дефект скрутки, фундаментальна мода.

Алексеев К. Н. Локализация фундаментальных мод на дефекте скрутки в скрученном анизотропном волокне / К. Н. Алексеев, Б. П. Лапин, М. А. Яворский // Ученые записки Таврического национального университета имени В. И. Вернадского. Серия : Физико-математические науки. – 2014. – Т. 27 (66), № 2. – С. 5-13.
В данной статье изучено влияние дефекта скрутки на распространение циркулярно поляризованной гауссовой моды в брэгговском скрученном слабонаправляющем волокне с одноосной материальной анизотропией. Было показано, что гауссова мода с правой круговой поляризацией в узком спектральном диапазоне внутри запрещённой зоны почти полностью проходит через волокно. Данный процесс сопровождается появлением локализованных на дефекте фундаментальных мод.
Ключевые слова: скрученное анизотропное волокна, дефект скрутки, фундаментальная мода.

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